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Diversity Block Code: A Trade-off for Multiplexing and Diversity Gains

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Abstract: Multiple antennas at both transmitter and receiver have been proposed in the literature to obtain spatial multiplexing or diversity based on the system requirement. Analytical approaches have been found in the literature, suggesting obtainable trade-off between multiplexing and diversity. However, there exists no implementable demonstration to achieve the same. In this study, a generator polynomial for a block code namely diversity block code has been proposed that demonstrates achievable trade-off between multiplexing and diversity. It is found that at low signal to noise ratio, the proposed codes provides the best bit error rate performance in comparison to its counterparts, systems with spatial multiplexing and transmit diversity. Beyond 7dB SNR, the system with the proposed code is being superior to the system with spatial multiplexing.

Key words: Multiplexing gain, diversity gain, spatial multiplexing, space-time block codes, diversity block codes

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INTRODUCTION

Radio communication with Multiple Input Multiple Output (MIMO) system is adopted in the wireless communications (Foschini and Gans, 1998). Instead of using single transmit and receive antenna pair, MIMO system uses an array of antennas both at transmitter and receiver (Foschini and Gans, 1998). In general, MIMO system utilizes the spatial domain which remains unused by single antenna transmission methods. The major achievable attributes that leads to future wireless communication systems towards MIMO systems are its capability to provide higher data rates, small error rates and better co-channel interference mitigation at relatively lower signal-to-noise ratio (Gosalia and Lazzi, 2003). The basic principle involved in MIMO communication system is to transmit several input signals at the same time and over certain frequency band from different transmitting antennas. Multiple antennas are used at the receiver to receive transmitted signal vector from multiple antennas. The received signals are processed at receiver to detect and estimate the transmitted data stream which include spatial dimensionality over time.

The wireless communication channel suffers with signal degradation because of multi-path fading and its frequency selective nature. MIMO receiver can distinguish and detect signals based on multi path fading characteristics within certain scattering environment (Martone, 2002). Subsequently, with MIMO systems, it is possible to achieve higher data rates through multiplexing and obtain better quality of service through diversity. In response to this, significant amount of research has been conducted for MIMO wireless systems with major focuses onto either spatial multiplexing or spatial diversity.

DIVERSITY-MULTIPLEXING GAIN TRADE-OFF

As found in the existing literature (Foschini, 1996), (Alamouti, 1998), (Seshadri and Winters, 1993) and (Tarokh *et al*., 1998) multiplexing and diversity have their respective major focuses onto enhancing data rate and improving Bit Error Rate (BER) performance at the receiver. However, it may require obtaining a trade-off between such multiplexing and diversity in the context of MIMO system. Authors in (Grokop and Tse, 2009) have provided analytical basis in obtaining such trade-off. Assuming SNR to be input signal to noise ratio, R denoting data rate, P_e denotes probability of error, r denotes multiplexing gain and d denotes diversity condition as shown in Eq. 1 and 2:

$$\lim_{SNR\to\infty}\left(\frac{R}{\log(SNR)}\right) = r \tag{1}$$

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$$\lim_{SNR\to\infty}\left(\frac{\log P_e(SNR)}{\log(SNR)}\right) = -d \qquad (2)$$

The trade-off between the multiplexing gain and diversity gain is denoted by d(r), which is a non-increasing function (Jiang *et al*., 2011). Assuming N_t and N_r be the number of transmitter and receiver antennas respectively, the trade-off function is characterized as shown in Eq. 3:

$$d(r) = (N_r + N_t + 1)\left(1 - \frac{r}{N_t}\right) \qquad (3)$$

To the best of our knowledge, no dedicated coding model has been proposed to demonstrate and analyse the performance of MIMO system in achieving multiplexing and diversity trade-off. In this study, a generic block code is modelled, named as Diversity Block Code (DBC) to demonstrate the achievable trade-off between multiplexing and diversity. Subsequently a trade-off between the data rate and quality of service in MIMO system is obtained.

PROPOSED DIVERSITY BLOCK CODES

As the principle objective of the proposed DBC is to obtain a trade-off between spatial multiplexing and diversity within a MIMO system, the length of the code word denoted as n is assumed to be integer multiple of transmit antennas. Let k be the length of the message block and n = lN_t, where l is the positive integer and length. Similar to the existing block code (Lin and Costello, 2004) in literature, let's assume a message block $u = (u_1, u_2,..., u_k)$ encoded to code word vector $v = (v_1, v_2,..., v_n)$ as shown in Eq. 4:

$$v = u.G \qquad (4)$$

where, G is the generator matrix of dimension (k×n) To demonstrate the achievable trade-off with the proposed code, Alamouti's (Alamouti, 1998) simple transmit model of (2×2) MIMO system is considered, where 2 bits have been transmitted over 2-consecutive bit interval while considering spatial diversity. Within the same model, 4 bits have been transmitted in the same two consecutive transmission intervals; while achieving spatial multiplexing is the major objective (Foschini, 1996). To adapt with the same (2×2) MIMO model, the following new generator matrix of dimension (3×4) is adopted as shown in Eq. 5:

$$G = \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{pmatrix} \qquad (5)$$

Hence, the prospective length of the message block is k = 3 and the encoded code word length n = 4. Subsequently, the achievable transmission rate is 3/4 per transmit antenna whereas per antenna transmission rate in Alamouti's and spatial multiplexing schemes are 1/2 and 1, respectively.

Similar to any other Maximum Likelihood (ML) detection scheme, the adopted code word has 2^n possible candidates; there are only 2^k legitimate code words with minimum hamming distance $d_{min} = 2$. In general, for a rate:

$$\left(\frac{k}{n}\right)$$

linear block code, the minimum Hamming distance is upper bounded by the singleton bound as given in Eq. 6:

$$d_{min} \le n-k+1 \qquad (6)$$

It has to be noted that the encoded code word is obtained as shown in Eq. 7:

$$v_{(1\times n)} = \left(u_{(1\times k)} G_{(k\times n)}\right)_{mod-2} \qquad (7)$$

where, (mod-2) is the XOR operation. Assuming denotes the estimate of the code word possible error, the decoding is performed with the following Eq. 8:

$$\tilde{u}_{(1\times k)} = \left({}_{(1\times n)} G^{T}{}_{(n\times k)}\right)_{mod-2} \qquad (8)$$

It turns out to be single error correcting code as well as is capable of providing 25% higher spectral efficiency, compared to Alamouti's simple transmission scheme. Furthermore, due to the inherent error correcting capability the information block with the proposed transmission model is expected to perform better compared to (Foschini, 1996) and (Alamouti, 1998) at least at low signal to noise ratio.

PERFORMANCE EVALUATION ARCHITECTURE

To evaluate the robustness of the proposed DBC to demonstrate the capability of achieving trade-off between multiplexing and diversity gains the block diagram of the MIMO system model as shown in (Fig. 1) has been considered. Consider the MIMO channel with N_t transmitting and N_r receiving antennas. Let's assume an

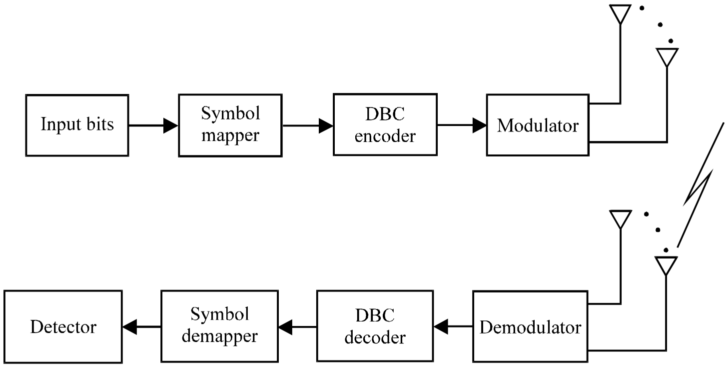

Fig. 1: Block Diagram of Diversity Block Code (DBC) for Multiple Input Multiple Output (MIMO) systems

information frame x of length L = kp where p is a positive integer number, which is mapped into a message matrix U of dimension (k×p) with $U = (u_1, u_2, ..., u_p)$ where $u(.) = (u_1, u_2, ..., u_k)^T$. Each of the message blocks u(.) is encoded to a code word v(.) with the proposed generator matrix, where $v(.) = (v_1, v_2, ..., v_n)^T$. Hence, the message matrix U is encoded to a matrix V of dimension (n×p). Each of the encoded vectors is mapped to $(N_t \times 1)$, resulting encoded signal matrix S of dimension $(N_t \times (1p))$ The received signal matrix is represented as shown in Eq. 9:

$$Y = HS + N \tag{9}$$

where, $y = (y_1, y_2, ..., y_{Ip})$ with $y(.) = (y_1, y_2, ..., y_{Nr})^T$, H is a rayleigh fading channel of size $(N_r \times N_t)$ in which every entry represents the gain of particular path and is denoted as h_{ij}, where $I \in \{1, 2, ..., N_r\}$, $j \in \{1, 2, ..., N_t\}$ the Gaussian noise matrix $n = (n_1, n_2, ..., n_{Ip})$, where $n(.) = (n_1, n_2, ..., n_{Nr})^T$ with zero mean and unity variance; transmitted signal matrix $S = (\breve{S}_1, \breve{S}_2, ..., \breve{S}_p)$ as shown in Eq. 10:

$$\breve{S}_{(.)} = \begin{pmatrix} v_{11} & \cdots & v_{1L} \\ \vdots & \ddots & \vdots \\ v_{Nt1} & \cdots & v_{NtL} \end{pmatrix} \tag{10}$$

The estimate of the received signal is as shown in Eq. 11:

$$\hat{S} = \arg\min_{SEC_i^p} \| Y - \sqrt{\frac{snr}{N_t}} HS \|^2 \tag{11}$$

where $\hat{S} = (\acute{S}_1, \acute{S}_2, ..., \acute{S}_p)$, $\acute{S}$ is the estimate of $\breve{S}$ and $\tilde{v}$ denoting estimated code word of dimension (n×1) that is obtained by reshaping $\acute{S}_p$ of size $(N_t \times L)$. Hence, estimated code word matrix is $\tilde{v} = [\tilde{v}_1, \tilde{v}_2, ..., \tilde{v}_p]^T$. The decoded information sequence is as shown in Eq. 12:

$$\tilde{U} = G.\tilde{v} \tag{12}$$

The probability of error defined in lower bound for (2×2) DBC is as given in the Eq. 13:

$$p(S, \hat{S} | Y) = \frac{1}{4} \exp - \left[\frac{(2\sigma N_t)^2 - d_{min}^2}{8\sigma^2} \right] \tag{13}$$

where, S is the input signal, $\hat{S}$ is the estimate of input signal, σ^2 is the noise variance, N_t is the number of transmitting antennas and d_{min} is the minimum Hamming distance.

PERFORMANCE EVALUATION

Here, the performance approximation of different parameters that had the significance on the performance of DBC is presented.

Let b = 1/n be the number of parity bits transmitted by each source separately. The condition for the successful decoding is obtained as in Eq. 14:

$$\frac{k}{n} \approx \frac{1}{2} \left(1 + \frac{P_c}{\sigma^2} \right) \tag{14}$$

where, P_c is the transmitted power. After the first stage of decoding, the resulting k bit sequence is expected to contain (kP/3) errors, where P is the probability of error. For successful decoding in the second stage with H as entropy as shown in Eq. 15:

$$kH\left(\frac{P}{3} \right) \approx \frac{b}{2} \log \left(1 + \frac{P_c}{\sigma^2} \right) \tag{15}$$

Using the fact that code rate $(k/n) = (k)/(N_t+2b)$, we obtained Eq. 16:

$$\frac{k}{n} = \left(\frac{2k}{L - N_t} \right) \tag{16}$$

From the Eq. 15 and 16, we obtained Eq. 17:

$$\left(\frac{2k}{n-N_t}\right)H\left(\frac{P}{3}\right)=\frac{1}{2}\log\left(1+\frac{P_c}{\sigma_c}\right) \tag{17}$$

Average power transmitted per symbol, P_{avg} can be obtained as in Eq. 18:

$$P_{avg}=P_c\left(\frac{N_t+2b}{n}\right) \tag{18}$$

SIMULATION RESULTS

To evaluate the multiplexing and diversity trade-off with the adopted transmission model, a MIMO system with $N_t = 2$ and $N_r = 2$ is considered. An information frame of (30×10^3) bits has been transmitted assuming $k = 3$. Subsequently, encoded code length at the modulator is found to be $n = 4$. Two different modulation schemes have been considered; Binary Phase Shift Keying (BPSK) and Quadrature Phase Shift Keying (QPSK), for simplicity. Figure 2 representing the receiver BER performance comparison of the MIMO system exploiting the proposed DBC considering the MIMO system with spatial multiplexing as well as with spatial diversity independently. At low SNR till 7dB, simulation result shows that the system with the proposed DBC is superior to the system without DBC while exploiting spatial multiplexing and diversity. This dominance is consistent for both BPSK and QPSK modulation schemes, which is due to the inherent coding gain. However, beyond 7dB, spatial diversity started to dominate the BER performance, while the system with the proposed DBC continued to dominate the system with only spatial multiplexing. There is 2dB SNR gain obtained with the proposed DBC over spatial multiplexing, whereas the bit transmission rate remains 0.5, 0.75 and 1 per antenna per time slot for the system with spatial diversity, DBC and only with spatial multiplexing, respectively. Figure 3 represents the MIMO system, in comparison of transmitted average power exploiting the proposed DBC performance with spatial multiplexing and spatial diversity independently. The simulation result shows that the proposed DBC confirms the expected trade-off between spatial multiplexing and diversity. It is observed that 1dB SNR gain obtained with DBC over spatial diversity. Even though the spatial multiplexing dominated the proposed system with DBC, the system provided a trade-off between spatial multiplexing and spatial diversity. Simulation results

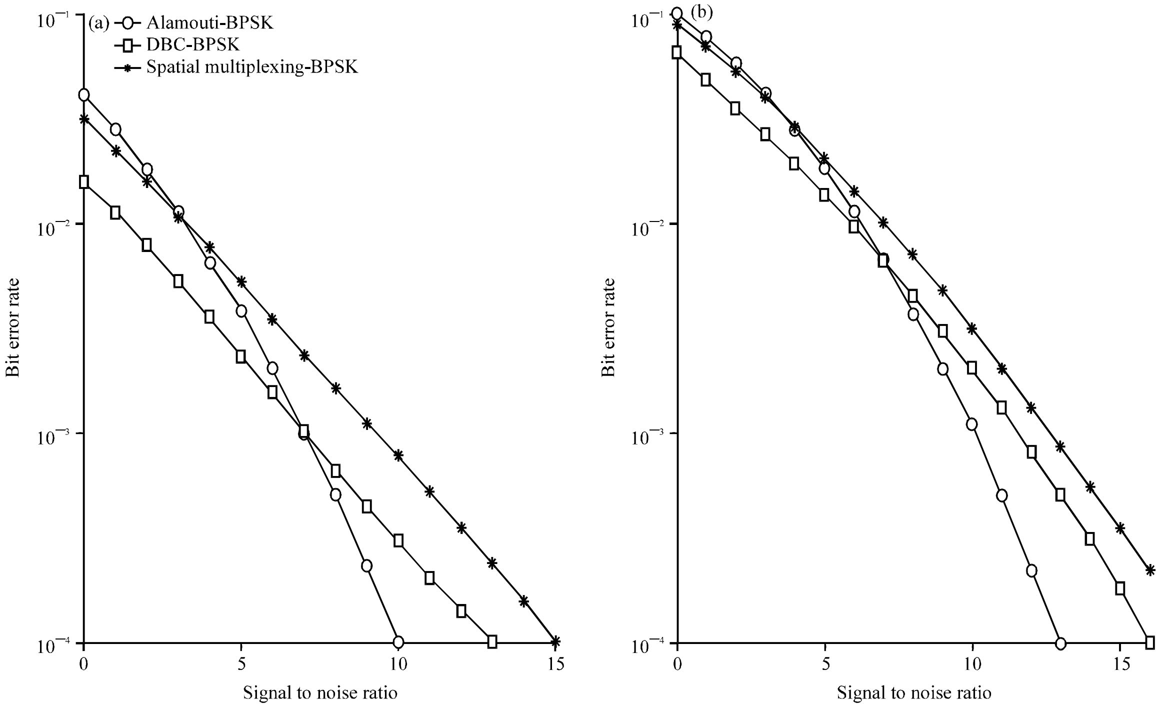

Fig. 2(a-b): Bit Error Rate (BER) performance comparison of Diversity Block Code (DBC) to achieve trade-off between diversity and spatial multiplexing, (a) Binary Phase Shift Keying (BPSK) Modulation and (b) Quadrature Phase Shift Keying (QPSK) Modulation

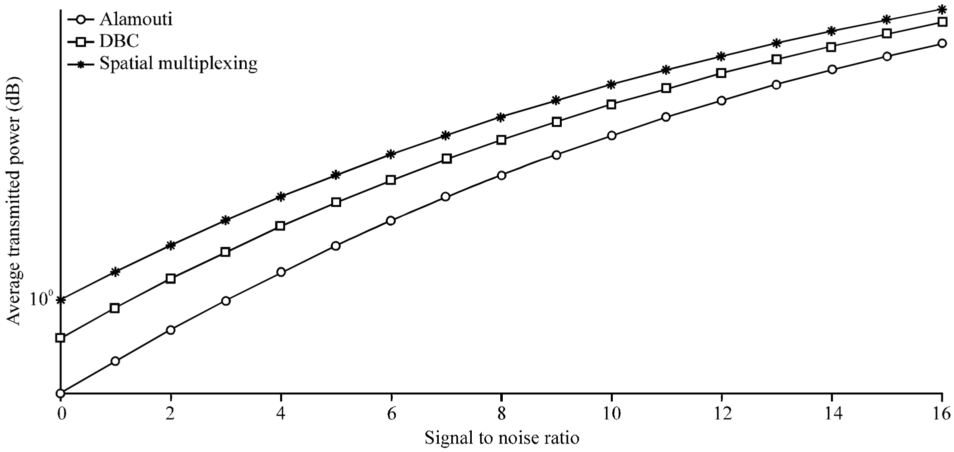

Fig. 3: Performance comparison of Diversity Block Code (DBC) for average power transmitted to obtain trade-off between diversity and spatial multiplexing

presented in Fig. 2 and 3 demonstrate the achievable trade-off between the spatial multiplexing and spatial diversity, exploiting the proposed DBC.

CONCLUSION

A new generator matrix for block code has been adopted to obtain a trade-off between spatial multiplexing and spatial diversity within a MIMO system. It has been observed that the system with the proposed code provides the best BER performance at low SNR compared to the systems with spatial multiplexing (Foschini, 1996) and transmit diversity (Alamouti, 1998). Such performance superiority of the system with the proposed code over spatially multiplexed system continues even at higher SNR. It is witnessed that the proposed system is dominating the spatial diversity in terms of the average power required to be transmitted for a given BER and provides with an easily adoptable trade-off between spatial multiplexing and transmit diversity. The investigation onto higher order generator polynomial that provides higher d_{min} which is expected to be suitable for higher order modulation scheme as well as higher order MIMO system is the future focus of the proposed study.

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