Advice on Exploratory Factor Analysis

Introduction

Exploratory Factor Analysis (EFA) is a process which can be carried out in SPSS to validate scales of **items** in a questionnaire. The purpose of an EFA is to describe a multidimensional data set using fewer variables. Once a questionnaire has been validated, another process called Confirmatory Factor Analysis can be used. This is supported by AMOS, a 'sister' package to SPSS.

There are two forms of EFA known as Factor Analysis (FA) and Principal Component Analysis (PCA). The reduced dimensions produced by a FA are known as **factors** whereas those produced by a PCA are known as **components**. PCA will always work but FA may not converge to a solution.

FA analyses the relationship between the individual item variances and common variances shared between items whereas the PCA analyses the relationships between the individual item variances and total (both common and error) variances shared between items. FA is therefore preferable to PCA in the early stages of an analysis as it allows you to measure the ratio of an item's unique variance to its shared variance, known as its **communality**. As dimension reduction techniques seek to identify items with a shared variance, it is advisable to remove any item with a communality score less than 0.2 (Child, 2006). Items with low communality scores may indicate additional factors which could be explored in further studies by developing and measuring additional items (Costello and Osborne, 2005).

There are different **EFA methods**. If you are only dealing with your sample for further analysis (i.e. it is a population in terms of the EFA) it is advisable to use the **Principal Axis Factoring** method. Otherwise, if you are trying to develop and instrument to be used with other data sets in the future, it is advisable to use a sample-based EFA method such as **Maximum Likelihood** or **Kaiser's alpha factoring** (Field, 2013: 674-675).

Whether to **rotate the factors** and the type of rotation used also needs to be decided. An **orthogonal rotation** can improve the solution from the unrotated one but it forces the factors to be independent of each other. The most popular orthogonal rotation technique is **varimax**. An **oblique rotation** allows a degree of correlation between the factors in order to improve the intercorrelation between the items within the factors. Although Reise et al. (2000) give several reasons why it should be considered, it is more difficult to interpret so it advised that it should only be considered if the orthogonal solution is unacceptable. Field (2013: 681) recommends using either the **direct oblimin** or **promax** rotation with the default parameter settings. An oblique rotation creates two additional factor matrices called **pattern** and **structure**. It is the pattern matrix which needs to be analysed in the same way as the single rotated factor matrix obtained from orthogonal rotations.

Each item is given a score for each factor. Following the advice of Field (2013: 692) we recommend **suppressing factor loadings less than 0.3**. Any item with all scores suppressed should be removed. Scores greater than 0.4 are considered stable (Guadagnoli and Velicer, 1988). Items should not cross-load too highly between factors (measured by the ratio of loadings being greater than 75%). There should be as many factors as possible with **at least 3 non-cross-loading items with an acceptable loading score**. Items should be removed one by one until the solution satisfies all the requirements. The number of extracted factors may need to be reduced during the process.

After the EFA has been carried out there a **validation process**. There are different ways to extract and double-check the derived scales. For a successful analysis there should be **a higher average correlation between the items in the derived scales than the average correlation between the scales**. The proportion of the total variance explained by the retained factors should be noted. **As a general rule this should be at least 50%** (Streiner, 1994). The adequacy of the sample size should also be checked. The average communality should be checked for small samples. Finally, a test for multicollinearity based on the size of the determinant of the correlation matrix should be carried out.

Step by step approach

1. Before carrying out an EFA the values of the bivariate correlation matrix of all items should be analysed. It is easier to do this in Excel. High values are an indication of multicollinearity, although they are not a necessary condition (see Rockwell, 1975). Field (2013: 686) suggests removing one of a pair of items with bivariate correlation scores greater than 0.8. There is no statistical means for deciding which item of a pair to remove – this should be based on a qualitative interpretation.
2. Decide on the appropriate method and rotation (probably varimax to start with) and run the analysis.
3. Remove any items with communalities less than 0.2 and re-run.
4. Optimize the number of factors – the default number in SPSS is given by Kaiser's criterion (eigenvalue > 1) which often tends to be too high. You are looking for as many factors as possible with at least 3 items with a loading greater than 0.4 and a low cross-loading. Fix the number of factors to extract and re-run.
5. Remove any items with no factor loadings > 0.3 and re-run.
6. Remove any items with cross-loadings > 75% starting with the one with the lowest absolute maximum loading on all the factors and re-run.
7. Once the solution has stabilized, check the average within and between factor correlations. To obtain the factors, use a PCA with the identified items and save the regression scores. If there is not an acceptable difference between the within and between factor average correlations, try an oblique rotation instead.
8. Provided the average within factor correlation is now higher than the average between factor correlation, a number of final checks should be made:
    a. Check that the proportion of the total variance explained by the retained factors is at least 50%.
    b. Check the adequacy of the sample size using the KMO statistic. A minimum acceptable score for this test is 0.5 (Kaiser, 1974).
    c. If the sample size is less than 300 check the average communality of the retained items. An average value above 0.6 is acceptable for samples less than 100, an average value between 0.5 and 0.6 is acceptable for sample sizes between 100 and 200 (MacCallum et al., 1999).
    d. The determinant of the correlation matrix should be greater than 0.00001 (Field, 2013: 686). A lower score might indicate that groups of three or more questions have high intercorrelations, so the threshold for item removal should be reduced until this condition is satisfied.
    e. The Cronbach's alpha coefficient for each scale can also be calculated.
9. If the goal of the analysis is to create scales of unique items then the meaning of the group of unique items which load on each factor should be interpreted to give each factor a meaningful name.

Worked example

171 business men and women responded to a questionnaire on entrepreneurship which was constructed from 8 groups of questions derived from existing questionnaires, comprising of a total of 39 questions. Each of the questions comprised of a five point Likert response scale. As the data from the questionnaire was to be used in a further analysis it was decided to carry out an Exploratory Factor Analysis using the Principal Axis Factoring technique and a Varimax rotation.

A Pearson bivariate correlation of all the items was carried out in Excel. A conditional formatting was set for any correlations with an absolute value greater than 0.8.

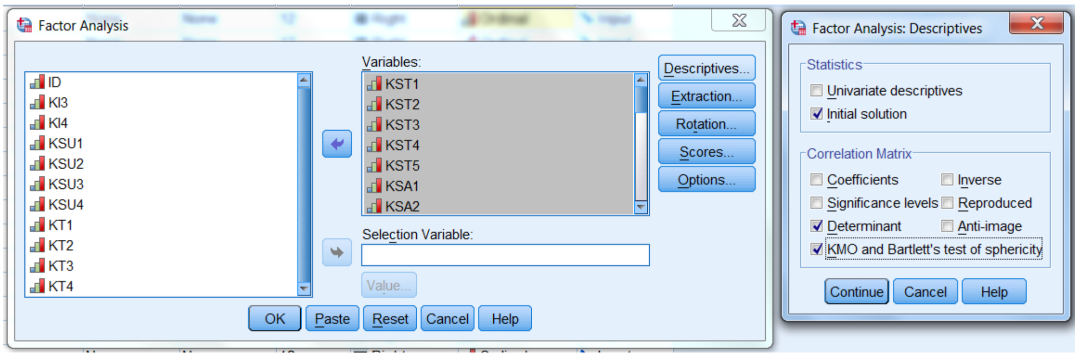
	KST1	KST2	KST3	KST4	KST5	KSA1	KSA2	KSA3	KSA4	KSA5	KSA6	KSA7	KSA8
KST1	1.000												
KST2	0.278	1.000											
KST3	0.118	0.433	1.000										
KST4	-0.138	0.267	0.392	1.000									
KST5	0.034	0.262	0.572	0.513	1.000								
KSA1	-0.029	0.192	0.489	0.288	0.400	1.000							
KSA2	0.025	0.319	0.304	0.336	0.185	0.467	1.000						
KSA3	0.182	0.536	0.434	0.275	0.430	0.272	0.261	1.000					
KSA4	0.127	0.266	0.409	0.304	0.552	0.266	0.139	0.369	1.000				
KSA5	0.145	0.281	0.452	0.196	0.251	0.354	0.278	0.359	0.209	1.000			
KSA6	0.309	0.101	0.135	0.002	0.074	0.072	0.068	0.065	0.128	0.173	1.000		
KSA7	0.266	-0.018	0.025	-0.214	-0.047	-0.098	-0.090	-0.017	0.032	0.154	0.332	1.000	
KSA8	0.146	0.148	0.306	0.154	0.283	0.473	0.235	0.138	0.283	0.330	0.187	-0.005	1.000
KL1	0.289	0.342	0.225	0.080	0.154	0.114	0.117	0.303	0.324	0.414	0.215	0.216	0.223
KL2	0.078	0.069	0.164	0.026	0.147	0.137	0.155	0.031	0.046	0.216	0.201	0.403	0.400
KL3	0.288	0.234	0.301	0.140	0.377	0.292	0.222	0.301	0.179	0.290	0.234	0.246	0.337
KL4	0.040	0.208	0.330	0.459	0.323	0.417	0.373	0.205	0.151	0.273	0.161	-0.089	0.406
KM1	0.334	0.281	0.168	0.107	0.157	0.154	-0.031	0.226	0.314	0.184	0.219	0.123	0.262
KM2	0.286	0.247	0.259	0.180	0.185	0.328	0.178	0.272	0.231	0.404	0.400	0.146	0.341
KM3	0.212	0.267	0.411	0.138	0.190	0.324	0.197	0.177	0.192	0.369	0.211	0.096	0.465
KM4	0.141	0.260	0.175	0.006	-0.010	0.239	0.327	0.108	0.203	0.364	0.153	0.074	0.501
KM5	0.038	0.118	0.313	0.160	0.261	0.421	0.298	0.159	0.189	0.406	0.162	-0.068	0.585
KM6	0.042	0.221	0.301	0.248	0.212	0.383	0.172	0.316	0.166	0.396	0.085	0.045	0.382
KSB1	0.180	0.352	0.155	0.131	-0.071	0.161	0.171	0.186	0.078	0.352	0.114	0.021	0.134
KSB2	0.203	0.337	0.081	0.104	0.153	0.185	0.223	0.314	0.117	0.163	0.139	0.141	0.249
KSB3	-0.001	0.314	0.247	0.264	0.309	0.366	0.323	0.253	0.140	0.220	0.031	0.045	0.486
KSB4	0.169	0.089	0.134	0.190	0.242	0.054	-0.020	0.081	0.360	0.174	0.212	0.122	0.067
KI1	0.223	0.062	0.381	0.291	0.383	0.243	0.025	0.135	0.328	0.441	0.275	0.070	0.338
KI2	0.077	0.061	0.527	0.303	0.474	0.477	0.196	0.137	0.248	0.420	0.195	0.003	0.517
KI3	0.178	0.226	0.396	0.143	0.177	0.313	0.202	0.165	0.197	0.363	0.200	0.060	0.452
KI4	0.135	0.265	0.151	-0.006	-0.040	0.250	0.320	0.110	0.172	0.375	0.129	0.074	0.469
KSU1	0.041	0.142	0.326	0.172	0.275	0.433	0.285	0.152	0.196	0.414	0.167	-0.069	0.559
KSU2	0.036	0.220	0.295	0.263	0.202	0.392	0.173	0.322	0.182	0.412	0.087	0.047	0.385
KSU3	0.172	0.356	0.159	0.128	-0.060	0.152	0.172	0.182	0.071	0.353	0.115	0.009	0.133
KSU4	0.153	0.298	0.032	0.099	0.133	0.149	0.212	0.317	0.087	0.120	0.101	0.099	0.196
KT1	-0.054	0.278	0.210	0.265	0.297	0.340	0.316	0.252	0.114	0.184	-0.007	0.004	0.447
KT2	0.132	0.042	0.095	0.194	0.242	0.025	-0.057	0.102	0.364	0.133	0.171	0.068	0.013
KT3	0.141	-0.016	0.317	0.292	0.361	0.183	0.013	0.147	0.305	0.374	0.193	-0.027	0.247
KT4	0.000	-0.002	0.469	0.288	0.464	0.431	0.173	0.122	0.207	0.366	0.141	-0.062	0.447

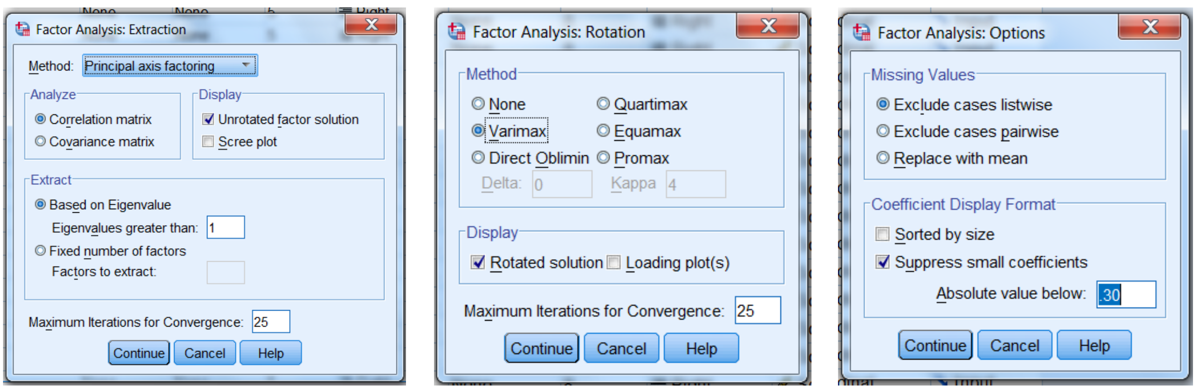
	KL1	KL2	KL3	KL4	KM1	KM2	KM3	KM4	KM5	KM6	KSB1	KSB2	KSB3	KSB4
KL1	1.000													
KL2	0.210	1.000												
KL3	0.204	0.435	1.000											
KL4	0.233	0.312	0.410	1.000										
KM1	0.406	0.115	0.262	0.267	1.000									
KM2	0.334	0.219	0.399	0.366	0.497	1.000								
KM3	0.380	0.344	0.323	0.413	0.362	0.397	1.000							
KM4	0.459	0.244	0.143	0.330	0.425	0.425	0.538	1.000						
KM5	0.183	0.256	0.288	0.379	0.194	0.495	0.332	0.392	1.000					
KM6	0.258	0.125	0.277	0.348	0.298	0.425	0.482	0.239	0.479	1.000				
KSB1	0.447	0.033	0.025	0.219	0.376	0.271	0.440	0.417	0.151	0.465	1.000			
KSB2	0.326	0.113	0.372	0.293	0.350	0.236	0.138	0.282	0.234	0.313	0.301	1.000		
KSB3	0.200	0.287	0.537	0.437	0.216	0.295	0.270	0.324	0.426	0.354	0.264	0.500	1.000	
KSB4	0.278	-0.009	0.086	0.208	0.316	0.238	0.140	0.221	0.181	0.225	0.229	0.168	0.018	1.000
KI1	0.219	0.233	0.373	0.355	0.302	0.481	0.308	0.163	0.238	0.309	0.245	0.205	0.218	0.403
KI2	0.182	0.290	0.325	0.499	0.153	[illegible]	0.343	[illegible]	0.487	0.347	0.136	0.228	0.278	0.202
KI3	0.348	0.280	0.266	0.381	0.306	0.345	0.971	0.527	0.331	[illegible]	0.427	0.086	0.243	0.108
KI4	0.413	0.216	0.116	0.284	0.361	[illegible]	0.528	0.937	0.356	0.237	[illegible]	0.259	0.317	0.149
KSU1	0.161	0.238	0.295	0.387	0.178	0.487	0.321	0.359	0.980	0.514	0.173	[illegible]	0.436	0.190
KSU2	0.266	0.134	0.282	0.340	0.269	0.387	[illegible]	0.227	0.470	0.983	0.456	0.318	[illegible]	0.223
KSU3	0.448	0.020	0.026	0.220	0.368	0.262	0.443	0.418	0.150	0.458	0.994	0.303	0.265	[illegible]
KSU4	0.266	0.045	0.321	0.237	0.293	0.175	0.082	0.228	0.197	0.282	0.265	0.978	0.461	0.109
KT1	0.141	0.234	0.497	0.392	0.159	0.243	0.225	0.277	0.399	0.327	0.230	0.458	0.983	-0.040
KT2	0.216	-0.091	0.020	0.161	0.280	0.197	0.096	0.163	0.139	0.195	0.191	0.108	-0.059	0.966
KT3	0.105	0.101	0.279	0.257	0.201	0.387	0.221	0.061	0.166	0.249	0.173	0.105	0.107	0.293
KT4	0.090	0.203	0.244	0.422	0.070	0.287	0.270	0.166	0.448	0.306	0.086	0.143	0.212	0.118

	KI1	KI2	KI3	KI4	KSU1	KSU2	KSU3	KSU4	KT1	KT2	KT3	KT4
KI1	1.000											
KI2	0.589	1.000										
KI3	0.271	0.331	1.000									
KI4	0.128	0.199	0.561	1.000								
KSU1	0.245	0.478	0.317	0.360	1.000							
KSU2	0.304	0.340	0.434	0.233	0.505	1.000						
KSU3	0.248	0.147	0.430	0.452	0.172	0.448	1.000					
KSU4	0.144	0.175	0.066	0.242	0.202	0.286	0.267	1.000				
KT1	0.164	0.232	0.231	0.303	0.408	0.319	0.231	0.457	1.000			
KT2	0.362	[illegible]	0.102	0.125	0.147	0.192	0.194	0.092	-0.080	1.000		
KT3	0.922	0.487	0.242	0.082	0.171	0.241	0.174	0.111	0.111	0.345	1.000	
KT4	0.513	0.955	[illegible]	0.169	0.438	0.288	0.096	0.148	0.219	0.124	0.502	1.000

This returned a table of correlations including 10 unique pairs of correlations with an absolute value greater than 0.8, with the lowest absolute value being 0.922. As this was markedly higher than the threshold it was decided to remove one item from each of these pairs based on a qualitative analysis of the items, leaving 29 items.

An EFA was then run on the remaining 29 items using a Principal Axis Factoring technique with a varimax rotation, providing the KMO statistics and determinant of the correlation matrix, retaining all factors with eigenvalues greater than 1 and suppressing all factor coefficients less than 0.3:





The communalities of the initial solution were observed. All were larger than 0.2 so all the items were retained.

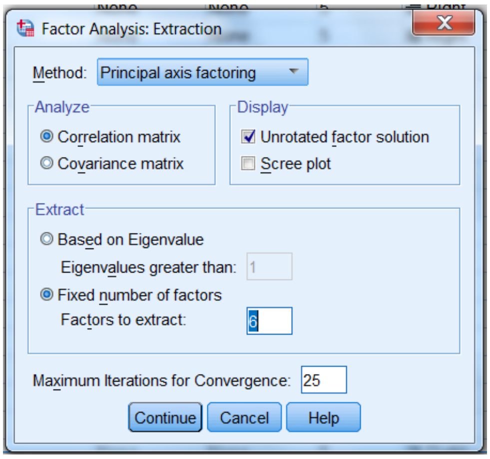
Rotated Factor Matrix[a]

	Factor					
	1	2	3	4	5	6
KST1				.463		
KST2		.606	.413			
KST3	.439	.672				
KST4		.442		-.302		
KST5	.305	.648			.390	
KSA1	.601	.331				
KSA2	.384	.328				
KSA3		.659				
KSA4		.465			.377	
KSA5	.427	.304	.325			
KSA6				.429		
KSA7				.660		
KSA8	.688					
KL1			.542			
KL2	.432			.485		
KL3	.356			.461		.489
KL4	.547					
KM1			.486		.364	
KM2	.388		.358		.310	
KM3	.493		.452			
KM4	.413		.610			
KM5	.649					
KM6	.420		.368			
KSB1			.704			
KSB2						.585
KSB3	.450					.659
KSB4					.566	
KI1	.381				.623	
KI2	.680				.367	

Extraction Method: Principal Axis Factoring.
Rotation Method: Varimax with Kaiser Normalization.

a. Rotation converged in 11 iterations.

This led to an initial solution comprising of 8 factors. However the 7th and 8th factors did not have 3 items with loadings > 0.4 in the rotated factor matrix so they were excluded and the analysis re-run to extract 6 factors only, giving the output shown on the left.



However, many items in the rotated factor matrix (highlighted) cross loaded on more than one factor at more than 75% or had a highest loading < 0.4. These were removed in turn, starting with the item whose highest loading was the lowest (KSA2) and the analysis re-run.

During the following analysis, in order that each factor had at least three items with loadings > 0.4, it was necessary to reduce the number of factors to 5, then to 4. This eventually yielded a stable solution after 13 steps with 18 items (see right). The item KM4 loaded on both Factor 1 and Factor 3 but the cross loading was < 75% so it was only included in the third scale.

Rotated Factor Matrix[a]

	Factor			
	1	2	3	4
KST3	.368	.612		
KST5		.818		
KSA1	.581	.349		
KSA3		.568		
KSA4		.583		
KSA7				.562
KSA8	.739			
KL1			.651	
KL2	.349			.601
KL3	.348			.601
KL4	.558			
KM1			.567	
KM4	.408		.630	
KM5	.676			
KSB1			.649	
KSB2			.422	
KSB3	.497			.304
KI2	.639	.316		

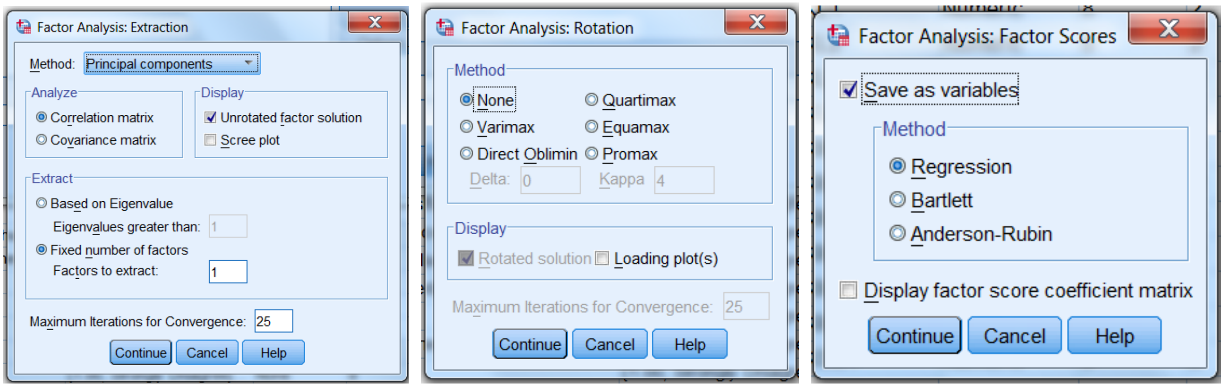
Extraction Method: Principal Axis Factoring.
Rotation Method: Varimax with Kaiser Normalization.

a. Rotation converged in 7 iterations.

The items loading on each factor were noted in order to create the trial scales.

Factor	Items
1	KSA1, KSA8, KL4, KM5, KSB3, KI2
2	KST3, KST5, KSA3, KSA4
3	KL1, KM1, KM4, KSB1, KSB2
4	KSA7, KL2, KL3

A PCA with a single factor was then run for each scale in turn as shown below. The Regression factor scores were saved.



The within scale correlations were calculated using Excel and the average scale correlations were calculated:

	B	C	D	E	F	G	H	I	J	K	L	M	N	O
1	KSA8	KL4	KM4	KM5	KSB3	KI2								
2	2	3	3	4	3	4			*KSA8*	*KL4*	*KM4*	*KM5*	*KSB3*	*KI2*
3	2	3	2	3	2	3		KSA8	1					
4	4	4	3	4	4	3		KL4	0.405526	1				
5	5	4	5	5	5	5		KM4	0.500944	0.33007	1			
6	3	4	3	4	4	3		KM5	0.585113	0.378957	0.391917	1		
7	5	5	5	5	5	5		KSB3	0.485861	0.436723	0.323537	0.425877	1	
8	2	5	2	4	5	4		KI2	0.516871	0.498922	0.245509	0.486598	0.277711	1
9	5	4	4	5	5	4								
10	4	4	4	5	3	4		Average	0.419342					
11	5	5	5	5	4	4								
12	3	5	3	4	4	4								

This yielded the following results:

Factor	**1**	**2**	**3**	**4**	**Overall**
Average within factor correlation	0.419	0.461	0.379	0.361	0.405

The regression scores for the scales were downloaded into Excel and a correlation analysis was run, yielding the results shown on the right.

	Factor1	*Factor2*	*Factor3*	*Factor4*
Factor1	1			
Factor2	0.476656	1		
Factor3	0.538243	0.298892	1	
Factor4	0.376263	0.22098	0.280408	1
Average	0.36524			

The average within factor correlation (0.405) was only slightly higher than the average between factor correlation (0.365). This was considered unacceptable as the within group correlations should have been considerable higher. An oblique factor rotation was then carried out.

A Principal Axis FA with a direct oblimin oblique rotation with Delta = 0 was carried out using the same 29 items as the original FA above. During the process of re-running the analysis the number of iterations for the Rotation was increased to 100 due to slow convergence.

A 4 factor solution eventually stabilized after 15 steps with 17 items as shown below. One item was removed for having communality < 0.2. KM4 was not included in Factor 1 because of its cross-loading on Factor 2 (even though this was < 75%).

Factor	Items
1	KSA1, KSA8, KL4, KM5, KI2
2	KL1, KM1, KM4, KSB1
3	KST3, KST4, KST5, KSA3, KSA4
4	KST1, KSA6, KSA7

The average within factor correlation was 0.404. The average between factor correlation was 0.276. This was a much better result than the orthogonal rotation and was considered acceptable.

Finally, validation checks were run. The KMO statistic was 0.819 (very good). The correlation matrix determinant was 0.002 (much higher than the critical value of 0.00001). The 4 factors explained 59.5% of the variation in the data, which was also acceptable.

The extracted communalities were exported into Excel and the average value was calculated (see below right). This was slightly lower than recommended for the sample size. According to MacCallum et al. (1999), for sample sizes between 100 and 200 it should be between 0.5 and 0.6. It was noted that the communailities of the three items on the fourth factor (highlighted) were all relatively low.

A PCA with a single component was carried out on the 4 scales in turn and the regression scores were saved. As a double, check a Cronbach's alpha reliability analysis was also run on each scale.

This yielded the following results:

Factor 1

Component Matrix[a]

	Component 1
KSA1	.732
KSA8	.795
KL4	.702
KM5	.762
KI2	.792

Eigenvalue = 2.869

Cronbach's alpha = 0.814

Factor 2

Component Matrix[a]

	Component 1
KL1	.773
KM1	.726
KM4	.768
KSB1	.742

Eigenvalue = 2.266

Cronbach's alpha = 0.744

Pattern Matrix[a]

	Factor 1	Factor 2	Factor 3	Factor 4
KST1				.497
KST3			.578	
KST4			.542	-.357
KST5			.855	
KSA1	.529			
KSA3			.563	
KSA4			.556	
KSA6				.467
KSA7				.597
KSA8	.776			
KL1		.585		
KL4	.458			
KM1		.527		
KM4	.401	.564		
KM5	.701			
KSB1		.702		
KSB2		.397		
KI2	.673			

Extraction Method: Principal Axis Factoring.
Rotation Method: Oblimin with Kaiser Normalization.

a. Rotation converged in 10 iterations.

Communalities

	Initial	Extraction
KST1	.277	.337
KST3	.518	.528
KST4	.453	.465
KST5	.621	.799
KSA1	.428	.450
KSA3	.350	.364
KSA4	.432	.394
KSA6	.219	.274
KSA7	.225	.348
KSA8	.544	.621
KL1	.426	.522
KL4	.463	.419
KM1	.362	.407
KM4	.512	.592
KM5	.427	.496
KSB1	.377	.440
KI2	.545	.574
Extraction Method: Principal Axis Factoring.		
	Average	.472

Factor 3

Component Matrix[a]

	Component 1
KST3	.773
KST4	.663
KST5	.851
KSA3	.667
KSA4	.717

Eigenvalue = 2.721

Cronbach's alpha = 0.787

Factor 4

Component Matrix[a]

	Component 1
KST1	.707
KSA6	.759
KSA7	.727

Eigenvalue = 1.606

Cronbach's alpha = 0.561

The first 3 scales have acceptable Cronbach's alpha scores, acceptable loadings on at least 4 items > 0.6 and acceptable eigenvalue sizes.

The low Cronbach's alpha score for the 4th scale is consistent with it only having 3 items with loadings > 0.6, its low eigenvalue and its low average communality, indicating that it should only be used with caution.

The scales should then be interpreted qualitatively and given an appropriate name (omitted).

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