Non-hierarchical signalling: two-stage financing game

(forthcoming in International Journal of Mathematics, Game Theory and Algebra)

Anton Miglo†, and Nikolay Zenkevich‡

† University of Guelph, Department of Economics, Guelph, Ontario, Canada, N1G 2W1, tel. (519) 824-4120, ext. 53054, e-mail: amiglo@uoguelph.ca.

‡ Saint-Petersburg State University, Faculty of Applied Mathematics and Control Processes, Universitetskii prospekt 35, Petergof, Saint-Petersburg, Russia 198504, tel. +7 (812) 428-71-59, e-mail: Zenkevich@isdgrus.ru.

2006

Abstract

The literature analyzing games where some players have private information about their "types" is usually based on the duality of "good" and "bad" types (GB approach), where "good" type denotes the type with better quality. In contrast, this paper analyzes a signalling game without types hierarchy. Different types have the same average qualities but different profiles of quality over time which are their private information. We apply this idea to analyze a financing-investment game where firms' insiders have private information about the firm's profit profile over time. If transporting cash between period is costless equilibrium is pooling with up-front equity financing. Otherwise equilibrium is either pooling with debt when the economy is stagnating, or separating when the economy is growing (some firms issue debt and some firms issue shares). This provides new theoretical results that cannot be explained by the standard GB models and which are consistent with some financial market phenomena.

Keywords: Asymmetric information, Non-hierarchical signalling, Financing, Debt-equity choice, Equilibrium refinements, Intuitive criterion, Mispricing

1 Introduction

The literature analyzing games where some players have private information about their "types" is usually based on the duality of "good" and "bad" types (GB approach), where "good" type denotes the type with better quality. Depending on the context, the quality could mean the quality of produced good, the ability to work etc. Typically in such a game, the "good" type tries to signal its type to uninformed players by sending the messages which cannot be mimicked by the "bad" type.[1] In contrast, this paper analyzes a signalling game without types hierarchy. Different types have the same average qualities but different performances profiles over time which are their private information. Hence a "good" type in the beginning may become "bad" in the end or "bad" in the beginning may become "good" in the end. We apply this idea to analyze a financing-investment game where firms' insiders have private information about the firm's profit profile over time.[2]

More specifically, we analyze a situation where a firm's initial shareholders have to raise funds for financing an investment project. There is no internal funds available and therefore the financing should be external. The cost of investment is known to the shareholders and to the potential investors while the expected profit is the shareholders' private information. Such a situation in a static context (one period) was well studied in the literature. The equilibrium is typically pooling where all firms issue debt which survive usual equilibrium refinements and which minimizes mispricing (undervaluation) for a "good" type, i. e. for a firm with high expected profit.[3] We thus consider a two-period situation. In each period there is an investment and a profit. As was noted previously we suppose that different types have the same average profit but different profit profiles over time which are their private information. Also, we assume that managers have the choice between issuing debt or equity. The solution of the game we obtained shares with the standard models the existence of pooling equilibrium with debt. However, in our game a separating equilibrium (which is efficient by definition) may exist as well. Which equilibrium prevails depends on the initial distribution of types in the economy.

To provide basic ideas about how the private information about firms profit profile over time can affect financing choice let us suppose that there are only two types of firms. One is "performance-improving" (I) and have an increasing expected profit, while others are "stagnant" (S) and have a flatter or decreasing expected profit. In such an environment, prices can be affected by the "lemon" effect in both periods.[4] Intuitively, I would seem to have an informational advantage in the first period: because of lower profits in this period, this type of firm can capitalize on the adverse selection problem. On the other hand, in the second period the informational advantage passes to S. We show that I and S face very different incentives regarding financial decisions. The point is that, generally speaking, debt has a shorter maturity than equity, which has by definition infinite maturity. Thus, the price of first-period equity is type-independent due to the two-period maturity of equity (contrary to the one-period of debt) and to the fact that both types face the same total

[1] For a review of signalling games see, for instance, Fudenberg and Tirole (1990) or Petrosjan and Zenkevich (1996).

[2] In the similar spirit, some researchers assume that firms have the same average profit but different parameteres of risk which are their private information. For example, in the second part of Brennan and Kraus (1986) cash flows are ordered by mean-preserving spread condition. It is shown that optimal securities are neither convex nor concave in this case. In Brick, Frierman and Kim (1998) firms' profits have the same average value but different variances. The authors do obtain some results about firm dividend policy.

[3] See, for instance, Nachman and Noe (1994).

[4] We use the term "lemon" problem to describe a situation where private information leads to underpricing for a "good" type. See Akerloff (1970) for a classical example.

profit over the two periods. As a result, if I were to issue equity in the first period, they would always be mimicked by S, who stand to gain in the second period by being perceived as growing and, therefore, as expecting high profits in the second period. The implication is that I are at a disadvantage for equity issues in the first period. This is the main engine driving the results of this article.

While I would definitely prefer debt to equity, incentives for S depend on the macroeconomic situation or on the initial distribution of types in the economy. The idea here is that if the economy is growing—there are on average more performance-improving than stagnating firms—interest rates tend to be more suitable for I. In particular, first-period interest rates would be relatively high compared to those of the second period, because I is considered "bad" in the first period and "good" in the second. Given such an interest rate profile, we show that if S plays debt, it would be beneficial to creditors, but not to the firm. This is because the creditors benefit in the first period due to the high interest rates and to the fact that S does well at that period.

The rest of this paper is organized as follows. The basic model and some preliminary results are presented in Section 2. Sections 3 and 4 provide the analysis of two-type and multiple-type economies respectively. The conclusion is drawn in Section 5.

2 Model.

Consider a firm with two-stage investment project. In each stage $t = 1, 2$ an amount b has to be invested. In each stage the project can either be successful (with probability ρ_t) or unsuccessful (with probability $1 - \rho_t$). If the former is the case the revenue $\widetilde{R}_t$ equals 1 and if the latter is the case the revenue equals 0. Total expected revenue over both periods is then $\rho_1 + \rho_2$. Since all firms have the same total discounted profit, total revenue can be normalized to unity without loss of generality. Hence $\rho_1 + \rho_2 = 1$ and we write $\rho_1 = \theta$ and $\rho_2 = 1 - \theta$. Firms differ only through the parameter θ. We assume $b < 1/2$ with the θ's restricted to the interval $[b, 1 - b]$, which implies that the investment has non-negative profitability in each period, i. e. the expected profit is not smaller than the amount of investments in period one ($b \leq \theta$) and in period two ($b \leq 1 - \theta$). A firm has increasing expected revenues and profits if $\theta < 1/2$, the profit profile is flat or declining if $\theta = 1/2$ or $\theta > 1/2$ respectively. If we let θ be distributed according to the density $f(\theta)$, then the total (average) first-period revenue is:

$$y = \int_b^{1-b} \theta f(\theta)\, d\theta$$

Clearly, total second-period revenue is then $1 - y$. This means that the economy is growing (revenues and profits are increasing) if $y < 1/2$, and it is stagnant or declining if $y = 1/2$ or $y > 1/2$.

The firm's shareholders are responsible for capital structure choice, investments and profit distribution. The initial capital structure is 100% equity, with n shares outstanding. The firm maximizes wealth of initial shareholders, who we will call the entrepreneur. Let α_0 and β_0 denote the initial proportions of equity owned respectively by outsiders and entrepreneur and let α_t and β_t denote their proportions immediately after the financing and investment for stage t were done. Clearly, $\alpha_0 = 1$, $\beta_0 = 0$ and $\alpha_t + \beta_t = 1$ for all t.

There exists universal risk-neutrality in this economy. In addition, the competition among investors is perfect. Insider shareholders know the firm's type, but the investors do not. The distribution of types is common knowledge.

To finance the first stage, the firm may issue either debt (d) or equity (e).[5] In both cases, the firm gets amount b from the market, which is immediately invested. Holding free cash flow is costly. The reasons are well-known: "empire building" or inefficient investments and acquisitions, which spread the resources under the manager's control; increasing manager compensations or direct entrenchment; etc.[6] More specifically, we assume that any available free cash flow disappears immediately, producing useless loss for the shareholders.[7] Knowing this the shareholders will never keep the free cash that implies that any available first-period profit will be distributed as dividend. Thus, in the second period only debt financing is possible.[8,9] As in the standard literature in this field, we assume that the contract of debt is enforceable at no cost.

The sequence of events is illustrated in figure 1. We assume that the firm's type is revealed to initial insiders in the period 0. The investors are identical and we will call them simply the market. The market determines the prices of issued securities. Also the market observes the first-period capital structure choice. However, it does not observe the profit previously realized by the firm.

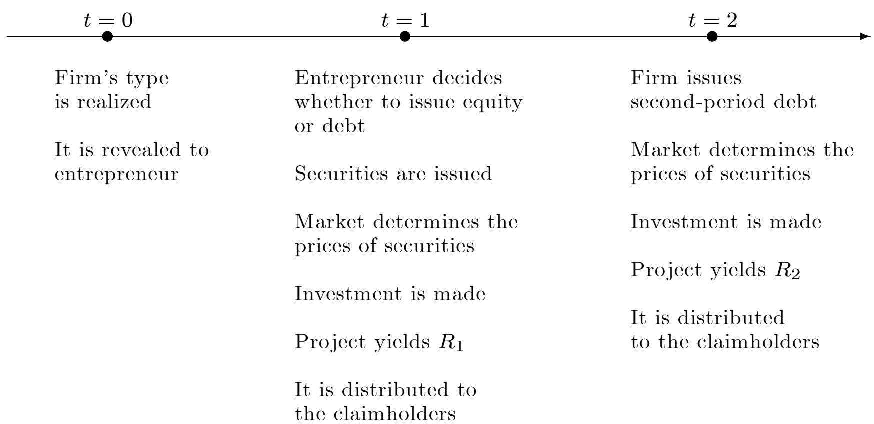

Figure 1. The sequence of events.

[5] More complicated securities are not considered here since the model's implications are all about equity-debt choice. Also, for the simplicity of exposition, we assume that only pure strategies can be played, although this is not crucial to the results.

[6] Easterbrook (1984), Jensen (1986). See also [7] and [14] for empirical analysis about the significance of manager's agency cost in holding cash

[7] We assume free cash to be any available cash at the end of a period, which means any resources that were not invested during the period or any received profits that were not used for interests or dividends.

[8] This is based on Myers and Majluf's (1984) idea that in one-period setting under asymmetric information, equity is never issued. Although our environments are quite different, one can show that the introduction of the possibility of equity issue in the second period does not alter any results.

[9] To simplify, we assume that mixed financing (debt/equity in the first period or cash/debt in the second) is not possible. The basic intuitions developed within this paper are not affected by introducing these possibilities. It is also important to note that the model can be extended by allowing mixed strategies (in game-theoretic terms), which can be interpreted to some extent as real mixed financing.

Throughout this article, we use the concept of Perfect-Bayesian equilibria (PBE) and also verify that off-equilibrium beliefs survive usual refinements like Cho and Kreps' (1987) intuitive criterion and consistency (Kreps and Wilson, 1982). The intuitive criterion seems to be not very powerful in games where pooling equilibrium is Pareto-efficient (see Cadsby, Frank, and Maksimovic, 1998). Fortunately, this is not the case in the present paper. In addition, note that perfect competition between outsiders implies zero market profit and risk-neutral valuation for any security issued. More specifically, we assume that there are at least two investors and the competition among them is in the Bertrand style (see Cho and Kreps (1987) or Nachman and Noe (1994)). Their pricing strategies are identical and equal to the expected value of the offered securities. Competition in the capital market therefore results in the price that yields zero net profit to investors.

From definition of debt and equity it follows that if debt is issued in period t with face value D then the debtholders expected payoff equals $E\left[\min(\widetilde{R}_t, D)\right]$. The shareholders are residual claimants. If new equity was issued the shareholders share the profit according to the number of shares owned.

2.1 Perfect market.

This subsection provides some useful information about benchmark pricing when the market knows the firm's type. Consider strategy e. Denote the issue of shares in period 1 by Δn, the price of issued shares by p_e^1 and the second period debt face value by D_e^2. The relations describing the pricing and the payoffs are:

1) first-period budget constraint:

$$b = p_e^1 \Delta n \tag{1}$$

2) market valuation of second-period debt:

$$b = E\left[\min(\widetilde{R}_2, D_e^2)\right] \tag{2}$$

3) market valuation of equity issued in the first period (recall that n denotes the initial number of shares):

$$p_e^1 = \frac{\overline{R}_1}{n + \Delta n} + \frac{E\left[\max(0, \widetilde{R}_2 - D_e^2)\right]}{n + \Delta n} \tag{3}$$

where $E[\widetilde{R}] = \overline{R}$. Given the identity:

$$\min(R, D) + \max(0, R - D) = R \tag{4}$$

and using equations (1) and (2), we can transform (3) to:

$$p_e^1 = \frac{\overline{R}_1 + \overline{R}_2 - 2b}{n}$$

Since $\overline{R}_1 + \overline{R}_2 = 1$, we get

$$p_e^1 = \frac{1 - 2b}{n} \tag{5}$$

Remark 1. p_e^1 depends only on the firm's total profit and not on its profit profile over time.

Using equation (2) and conducting a similar exercise for strategy d (for type θ), one can obtain the efficient (symmetric information) face values of debt (for the first and second period respectively):

$$D_d^1 = b/\theta, D_d^2 = b/(1-\theta) \tag{6}$$

If $\theta < 1/2$, the interest rate profiles in the case of d corresponding to type θ is downward sloping (and upward sloping if $\theta > 1/2$, respectively). Finally, note that regardless of how the investment is financed, the value of the firm for the entrepreneur is:

$$V = 1 - 2b \tag{7}$$

For example if e is played then the entrepreneur's expected payoff equals

$$\frac{n\overline{R}_1}{n+\Delta n} + \frac{nE\left[\max(0, \widetilde{R}_2 - D_e^2)\right]}{n+\Delta n}$$

Taken into account (3) and (5) this equals $1-2b$. As usual, in perfect market, the choice of financing does not matter.

2.2 Asymmetric information.

Now consider the situation where the firm's type is its private information. Let us introduce the payoff-functions. Denote by $V_j(\theta, \widehat{\theta})$ the entrepreneur's final payoff if the firm is of type θ but is perceived as type $\widehat{\theta}$, given the first-period action $j = e, d$. The following explains why the analysis of these functions is useful. Suppose that the market beliefs observing strategy j are characterized by a density function $\mu^j(\theta)$ with support $[b, 1-b]$.

Lemma 1. *Let the market beliefs observing strategy $j = e, d$ be μ^j. The pricing is then as if the market believes with probability 1 that the firm is type $\widehat{\theta}_j$, where*

$$\widehat{\theta}_j = \int \theta \mu^j(\theta) d\theta$$

Proof. Consider $j = d$. Let first-period debt face value equals D_d^1. The first-period lenders' expected first-period payoff is then: $\int D_d^1 \theta \mu^j(\theta) d\theta$. Risk-neutral valuation implies that it should be equal to b. Thus

$$D_d^1 = \frac{b}{\int \theta \mu^j(\theta) d\theta}$$

Analogously for second-period debt:

$$D_d^2 = \frac{b}{\int (1-\theta) \mu^j(\theta) d\theta}$$

Lemma 1 follows from (6). Now consider $j = e$. For second-period debt face value the reasoning is exactly as for D_d^2. Now consider the first-period share price. Since the firm's total expected profit equals 1 and since the second-period lenders' expected payoff equals b, the profit of insiders plus the profit of first-period outsiders of firm j equals $1-b$. Also, in the case of e, α_o^2 (that also shows the fraction of equity held by the first-period outsiders in the moment of first-period profit distribution) equals $\frac{\Delta n}{n+\Delta n}$. Thus, expected profit of first-period outsider shareholders is:

$$\frac{\Delta n}{n+\Delta n}(1-b)$$

Risk-neutral valuation implies that the expected revenue of first-period outsiders is b. Thus: $\frac{\Delta n}{n+\Delta n}(1-b) = b$ and $\Delta n = \frac{bn}{1-2b}$, which implies (because we know the budget constraint $p_e^1 \Delta n = b$) that

$$p_e^1 = \frac{1-2b}{n} \tag{8}$$

End proof.[10]

Note that under perfect information, the first-period share price equals $\frac{1-2b}{n}$, regardless of the issuer's type (equation (5)). The same result holds true under asymmetric information. It provides the intuition as to why growing firms prefer debt to equity—they cannot use their informational advantage in the first-period playing equity because the price is always the same.

Consider the features of functions $V_j(\theta, \widehat{\theta})$ for $j = e, d$. If the entrepreneur plays e

$$\alpha_c^2 = \frac{n}{n+\Delta n} = \frac{n}{n+b/p_e^1} = \frac{1-2b}{1-b}$$

as implied by (8). Thus

$$V_e(\theta, \widehat{\theta}) = \frac{1-2b}{1-b}(\theta + (1-\theta)(1 - \frac{b}{1-\widehat{\theta}})) \tag{9}$$

Also:

$$V_d(\theta, \widehat{\theta}) = \theta(1 - b/\widehat{\theta}) + (1-\theta)(1 - b/(1-\widehat{\theta})) \tag{10}$$

Obviously, $V_j(\theta, \theta) = 1-2b, j = e, d$, since this corresponds to complete information valuation. Observe also that $V_d(\theta, 1/2) = 1-2b$. The following properties are obvious:

Lemma 2. $\frac{\partial V_e(\theta,\widehat{\theta})}{\partial \theta} > 0$ *and* $\frac{\partial V_e(\theta,\widehat{\theta})}{\partial \widehat{\theta}} < 0$.

The idea behind Lemma 2 is that since the first-best share price in the first period is the same for all types, the types with high θ benefit from their informational advantages in the second period (when they are really "lemons"). On the other hand, a larger $\widehat{\theta}$ means a larger second-period interest rate, which is unprofitable.

Lemma 3. $V_d(\theta, \widehat{\theta}) \geq 1-2b$ *if and only if* $\theta \leq \widehat{\theta} \leq 1/2$ *or* $1/2 \leq \widehat{\theta} \leq \theta$. *Furthermore*

$$sign(\frac{\partial V_d(\theta, \widehat{\theta})}{\partial \theta}) = sign(\widehat{\theta} - 1/2) \tag{11}$$

[10] Note that the same result holds true if one introduces the possibility for second-period outsiders to observe first-period profit realization, given that market beliefs are Bayesian.

$$\widehat{\theta} < \min(\theta, 1/2) \Rightarrow \frac{\partial V_d(\theta, \widehat{\theta})}{\partial \widehat{\theta}} > 0 \tag{12}$$

$$\widehat{\theta} > \max(\theta, 1/2) \Rightarrow \frac{\partial V_d(\theta, \widehat{\theta})}{\partial \widehat{\theta}} < 0 \tag{13}$$

Proof.

$$V_d(\theta, \widehat{\theta}) \geq 1 - 2b \Leftrightarrow \frac{\theta}{\widehat{\theta}} + \frac{1-\theta}{1-\widehat{\theta}} \leq 2 \Leftrightarrow$$

$$\Phi(\widehat{\theta}) \equiv \theta(1-\widehat{\theta}) + (1-\theta)\widehat{\theta} - 2\widehat{\theta}(1-\widehat{\theta}) \leq 0,$$

where Φ is convex with roots $\widehat{\theta} = 1/2$ and $\widehat{\theta} = \theta$. This proves the first statement. The proof of (11) follows from:

$$\frac{\partial V_d(\theta, \widehat{\theta})}{\partial \theta} = -\frac{(1-2\widehat{\theta})b}{\widehat{\theta}(1-\widehat{\theta})}$$

To prove (12) and (13) one can check that

$$sign(\frac{\partial V_d(\theta, \widehat{\theta})}{\partial \widehat{\theta}}) = sign(\frac{\theta}{1-\theta} - (\frac{\widehat{\theta}}{1-\widehat{\theta}})^2)$$

Now $\widehat{\theta} < \min(\theta, 1/2)$ implies $\frac{\theta}{1-\theta} > (\frac{\widehat{\theta}}{1-\widehat{\theta}})^2$ while $\widehat{\theta} > \max(\theta, 1/2)$ implies $\frac{\theta}{1-\theta} < (\frac{\widehat{\theta}}{1-\widehat{\theta}})^2$. *End proof.*

Intuitively, by analogy with perfect information case, a downward sloping interest rates profile ($\widehat{\theta} \leq 1/2$) is suitable for growing firms, i. e. for firms with $\theta \leq \widehat{\theta}$ and not for firms with lower than average rate of growth ($\theta > \widehat{\theta}$), which are better off with upward sloping interest rate profile. Conversely for the case of stagnating economy ($1/2 \leq \widehat{\theta}$). The intuition behind condition (11) is the same. Now consider equation (12). If the interest rate profile is downward sloping then, for a firm that has lower than average rate of growth, making interest rate profile less upward sloping is profitable. On the other hand, if the interest rate profile is upward sloping then, for a firm with higher than average rate of growth, making the interest rate profile deeper is unprofitable (equation (13)).

Lemma 4. *$sign(V_d(\theta, \widehat{\theta}) - V_e(\theta, \widehat{\theta})) = sign(\widehat{\theta} - \theta)$.*

Proof. Consider $V_d(\theta, \widehat{\theta}) - V_e(\theta, \widehat{\theta})$. That is:

$$1 - b(\frac{\theta}{\widehat{\theta}} + \frac{1-\theta}{1-\widehat{\theta}}) - \frac{1-2b}{1-b}(\theta + (1-\theta)(1 - \frac{b}{1-\widehat{\theta}})) = \frac{(\widehat{\theta}-\theta)(1-b-\widehat{\theta})b}{(1-b)(1-\widehat{\theta})\widehat{\theta}} \tag{14}$$

The sign of the last expression depend obviously on the sign of $\widehat{\theta} - \theta$. *End proof.*

Figure 2 illustrates the first parts of Lemmas 2 and 3, condition (11) and Lemma 4.

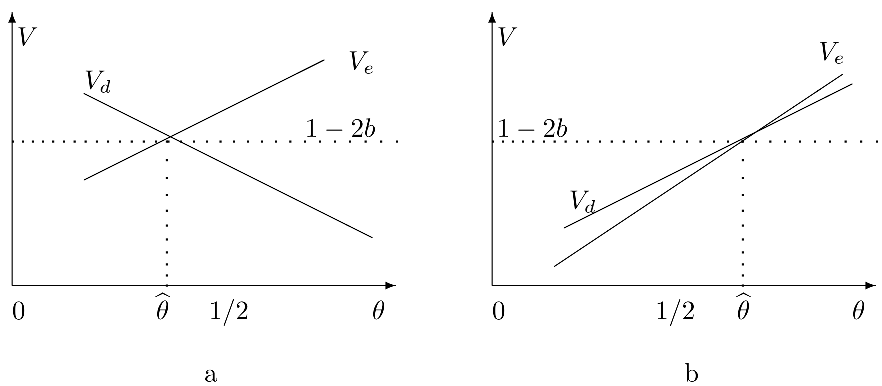

Figure 2. $V_d(\theta, \widehat{\theta})$ **and** $V_e(\theta, \widehat{\theta})$ **when: a)** $\widehat{\theta} < 1/2$**; b)** $\widehat{\theta} > 1/2$**.**

In both cases $V_e(\theta, \widehat{\theta})$ is increasing in θ. When $\widehat{\theta} < 1/2$ (Figure 2a), $V_d(\theta, \widehat{\theta})$ is downward sloping in θ and is upward sloping if $\widehat{\theta} > 1/2$ (Figure 2b). If the latter is the case the slope of $V_e(\theta, \widehat{\theta})$ is greater than that of $V_d(\theta, \widehat{\theta})$ meaning that the payoff from the strategy debt is less sensitive to adverse selection problem as compared to equity. This is in keeping with most of the literature in this field. Intuitively, if a firm is perceived as a less growing type than it is in reality then it will prefer debt to equity. This is because the second-period interest rate is the same in either case, but by playing debt, the firm gains in the first period by being a bad firm-type. If, in contrast, a firm is perceived by the market as a less growing than in reality type, it would prefer equity.

3 Two-type economy.

To generate the basic ideas, we first consider a two-type economy. Firm I is characterized by the parameter θ_I, firm S has parameter θ_S where $\theta_I < \theta_S$. By definition, S has better performance in the first period while I in the second (note that both may actually be declining, but S then declines faster). Let μ_0 be the proportion of type I firms, $0 < \mu_0 < 1$. Hence $y = \theta_I \mu_0 + \theta_S(1 - \mu_0)$. Since each firm may play two types of strategy (d or e), there are 4 potential candidates for equilibrium: two separating and two pooling. Given the concepts described in Section 2 a separating equilibrium is defined as follows:

1) type I plays j_I and type S plays $j_S \neq j_I, j_T \in \{e, d\}, T \in \{I, S\}$.

2) $V_{j_I}(\theta_I, \theta_I) \geq V_{j_S}(\theta_I, \theta_S)$ and $V_{j_S}(\theta_S, \theta_S) \geq V_{j_I}(\theta_S, \theta_I)$.

A pooling two-type equilibrium is defined as follows:

1) both type play j.

2) observing the strategy $j_{off} \neq j$ (off-equilibrium path) the market believes that the type is I with probability μ_I and the type is S with probability $\mu_S = 1 - \mu_I$ such that $V_j(\theta_I, y) \geq V_{j_{off}}(\theta_I, \theta_{\mu_p})$ and $V_j(\theta_S, y) \geq V_{j_{off}}(\theta_S, \theta_{\mu_p})$, where $\theta_{\mu_p} = \mu_I \theta_I + \mu_S \theta_S$.

3) If for type T $\max_\theta V_{j_{off}}(\theta_T, \theta) < V_j(\theta_T, y)$ then $\mu_T = 0, T \in \{I, S\}$.

The first condition means that different types play different strategies under separating equilibrium and the same strategy under pooling. The second condition represents the non-deviation condition for each type (individual rationality). Finally, the third condition in the case of pooling equilibrium assures that the equilibrium survives the intuitive criterion of Cho and Kreps (1987). This condition means that the market off-equilibrium beliefs are reasonable in the sense that if for any type T its maximal payoff from deviation is not greater than its equilibrium payoff then the market should place the probability 0 on possible deviations of this type. The definitions above are consistent with standard PBE definition (see, for instance, Fudenberg and Tirole, 1991) with an addition of intuitive criterion which is quite common in such kind of games (see, for instance, Nachman and Noe, 1994). Finally note that Lemma 1 insures that in described above equilibria the market makes zero-profit (competitive rationality).[11]

Proposition 1. *1) The situation where I plays e and S plays d is not an equilibrium; 2) if and only if $\theta_I \leq 1/2$, there exists a separating equilibrium where I plays d and S plays e.*

Proof. (i) *Part 1.* Suppose, in opposite, that such equilibrium exists. Of course, each type would have $1-2b$ in a separating equilibrium. From Lemma 2 $V_e(\theta_S, \theta_I) > 1-2b$ because $\theta_I < \theta_S$. Thus S would deviate from its equilibrium strategy to e and such equilibrium is impossible.

(ii) *Part 2.* Let $\theta_I \leq 1/2$. I does not mimic S. From Lemma 2 we have $V_e(\theta_I, \theta_S) \leq 1-2b$ because $\theta_I < \theta_S$. S does not mimic I. From Lemma 3, $V_d(\theta_S, \theta_I) \leq 1-2b$ because $\theta_I < 1/2$ and $\theta_I < \theta_S$. Now, if $\theta_I > 1/2$ then from Lemma 3 $V_d(\theta_S, \theta_I) > 1-2b$. Thus S would mimic I. *End proof.*

Intuitively, in the equilibrium described in Part 2, I does not deviate because by playing e it is not able to capitalize on its first-period informational advantage. The share's price in the first period does not depend on the firm's type (Lemma 1), while the interest rate in the second period will be unfavorable. S does not deviate because the interest rates profile is downward sloping or flat when $\theta_I \leq 1/2$, making d unprofitable for S (which performs better with upward sloping interest rates profile).

Proposition 2. *If and only if $y \geq 1/2$, pooling with d is an equilibrium.*

Proof. (i) *Part 1.* 1) Existence. Let $y \geq 1/2$. Consider pooling equilibrium where both types play d, which is supported by off-equilibrium market beliefs that the firm is S.[12] First of all, let us verify non-deviation for each type. Since $1/2 \leq y < \theta_S$, we gets from Lemma 3 $V_d(\theta_S, y) \geq 1-2b$. Thus the type S does not deviate. From Lemma 2, we have

$$V_e(\theta_I, y) \geq V_e(\theta_I, \theta_S) \tag{15}$$

The condition of non-deviation for the type I is obviously $V_d(\theta_I, y) \geq V_e(\theta_I, \theta_S)$. The latter follows from the condition (15) and Lemma 4:

$$V_d(\theta_I, y) \geq V_e(\theta_I, y) \geq V_e(\theta_I, \theta_S) \tag{16}$$

Let us now verify that off-equilibrium beliefs survive the intuitive criterion of Cho and Kreps (1987). To show this, let us calculate the maximal payoff of type S in the case that it

[11] Also note that by definition of pooling, the off-equilibrium beliefs are consistent (Kreps and Willson, 1982). If out of equilibrium path the market believes that the type is θ then it keeps the same beliefs in the second period (it follows from the definition of the payoff functions V). Otherwise the market off-equilibrium beliefs would be inconsistent.

[12] Note that in terms of the definition of pooling given above, we have here $j_{off} = e$, $\mu_S = 1$ and $\theta_p = \theta_S$.

plays e. Its payoff is evidently maximized if the market's belief places the probability 1 on type I observing equity, i. e. $V_e(\theta_S, \theta_I)$. If off-equilibrium beliefs survive intuitive criterion, this expression must be greater than $V_d(\theta_S, y)$.[13] It follows immediately from Lemmas 2 and 4:

$$V_e(\theta_S, \theta_I) \geq V_e(\theta_S, y) \geq V_d(\theta_S, y)$$

This completes the proof of sufficiency.

2) As for the necessary condition, if $y < 1/2$ then pooling with d is impossible because type S would deviate in e (from Lemma 3 its equilibrium payoff would be less than $1 - 2b$). *End proof.*

The idea behind the Proposition 2 is simple. Only if growing firms dominate the credit market ($y < 1/2$) will the interest rates profile be downward sloping, creating incentives for stagnating firms to play e.

Proposition 3. *1) if $y < 1/2$, pooling with e is not an equilibrium; 2) if $y \geq 1/2$ and if pooling with e exists, then mispricing is greater under that than under pooling with d.*

Proof: see Appendix 1.[14]

Intuitively, if y is low, then in the case of pooling with e second-period interest rate is low, making high profit for the type S. In some cases this profit is even greater than maximal possible profit under the strategy d. This situation is not an equilibrium because the market should set the probability 0 on the possibility for S to play d, making off-equilibrium interest rates suitable for the type I, that would deviate to d.[15] Secondly, if pooling with e exists, then mispricing is greater than it is under pooling with d. Intuitively by analogy with Lemma 4, type I (undervalued under pooling equilibrium, because S can always achieve at least first-best using e as a last resort) prefers pooling with debt over pooling with equity. Propositions 1, 2 and 3 are at the root of two major insights of this paper; they provide clues about the link between initial distribution of types in the economy and individual firm capital structure policy, and they show why debt is a signal of a firm's increasing performance while equity is a signal of decreasing performance.

The main conclusion of the above analysis is that performance-improving firms definitely prefer debt while stagnating firms base their strategy on the macroeconomic situation—if the economy is growing, they will issue equity, and if the economy is stagnating, both strategies can lead to equilibrium. Also note that in a two-type economy, separating equilibrium always dominates pooling by minimal mispricing. However, the intuition about the existence of pooling equilibria is useful and it will be further applied in Sections 4 and 5.

4 Multiple type economy.

To provide more ideas about the role of macroeconomic situation in this game, consider a multiple type economy, and suppose that $f(\theta) > 0, \forall\theta$ on the support $[b, 1-b]$. Here

[13] Otherwise μ_S should be equal to 0.

[14] We use the standard concept of mispricing that can be found, for example, in Nachman and Noe (1994). The magnitude of mispricing in a given equilibrium equals to that of undervalued types. For instance, if the strategy j is played by undervalued type T (the undervaluation is only possible in pooling) then the mispricng equals $1 - 2b - V_j(T, y)$. The overvaluation of overvalued type does not matter. Note that the total profit in the economy is always the same across different equilibriums.

[15] Remark that, since $y < 1/2$ implies $\theta_I < 1/2$, a separating equilibrium exists in this case.

again we consider two types of equilibria: pooling and semi-separating (or two-strategy equilibrium). A two-strategy equilibrium is defined as follows:

1) There are two sets Θ_e and Θ_d such that any $\theta \in \Theta_e$ plays e and any $\theta \in \Theta_d$ plays d, $\Theta_e \cap \Theta_d = O$, $\Theta_e \cup \Theta_d = [b, 1-b]$ and the measure of probability of O equals 0.

2) $V_e(\theta, \theta^e) \geq V_d(\theta, \theta^d), \forall \theta \in \Theta_e$ and $V_d(\theta, \theta^d) \geq V_d(\theta, \theta^e)$, where $\theta^e = \frac{1}{M(\Theta_e)} \int_{\theta \in \Theta_e} \theta f(\theta) d\theta$ and $\theta^d = \frac{1}{M(\Theta_d)} \int_{\theta \in \Theta_d} \theta f(\theta) d\theta$.

Note that $M(\Theta_j)$ is the measure of probability of Θ_j and θ^j denotes the expected type of firms playing $j \in \{e, d\}$. A pooling equilibrium is defined as follows:

1) all types play j.

2) observing the strategy $j_{off} \neq j$ (off-equilibrium path) the market beliefs are $\mu^{j_{off}}$ such that $V_j(\theta, y) \geq V_{j_{off}}(\theta, \theta_{\mu^{j_{off}}}), \forall \theta$, where $\theta_{\mu^{j_{off}}} = \int \theta \mu^{j_{off}}(\theta) d\theta$.

3) if for type θ $\max_\mu V_{j_{off}}(\theta, \theta_\mu) < V_j(\theta, y)$ then $\mu^{j_{off}}(\theta) = 0$, where $\theta_\mu = \int \theta \mu(\theta) d\theta$.

In each case the first condition means that both strategies are played in two-strategy equilibrium and that all types play the same strategy under pooling. The second condition represents the non-deviation condition for each type (individual rationality). Finally, the third condition in the case of pooling equilibrium assures that the equilibrium survives the intuitive criterion of Cho and Kreps (1987).

Proposition 4. *1) In any two-strategy equilibrium, there exists $\theta^* \in (b, 1-b)$ such that firms with $\theta > \theta^*$ play e and firms with $\theta < \theta^*$ play d; 2) If $y \leq 1/2$ (non-declining economy), this equilibrium exists.*

(i) *Proof of part 1.* Since both strategies are played, the $V_e(\theta, \theta^e)$ and $V_d(\theta, \theta^d)$ must intersect (otherwise we would have a pooling equilibrium like in Figure 3b) and the intersection is unique since the payoffs are linear in θ, as shown in the foregoing discussion.[16]

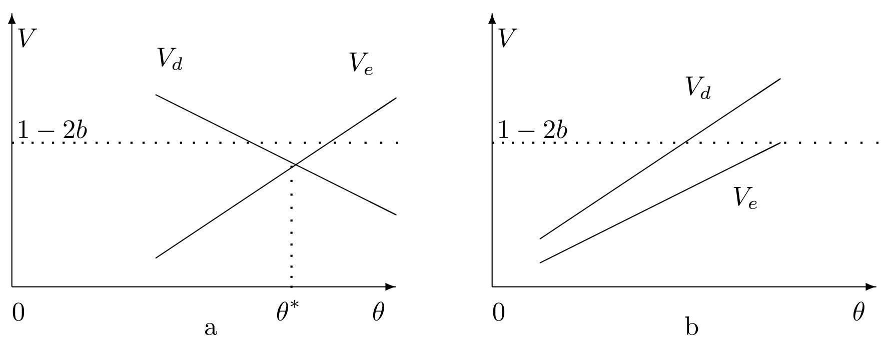

Figure 3. "Separating" and pooling equilibrium in multiple type economy.

Thus, the only candidates for a two-strategies equilibrium are either the one described in the Proposition or one where firms with $\theta > \theta^*$ issue debt and firms with $\theta < \theta^*$ issue equity. If we suppose that the latter is the case, contrary to the Proposition, it is only possible if V_d is increasing and cuts V_e from below. But the interest rates combination supporting this

[16] Recall that θ^j is the expected type of firms playing $j = e, d$.

situation is impossible. First of all, it must be that $1-2b > V_e(\theta^*, \theta^e) = V_d(\theta^*, \theta^d)$. If it does not, all firms playing debt ($\theta > \theta^*$) have more than first-best, which contradicts risk-neutral valuation. But in this case equity market makes positive profit (all issuing equity firms have less than $1-2b$) which contradicts equilibrium's concept. The only possible equilibrium is in Figure 3a where firms with $\theta > \theta^*$ issue equity and firms with $\theta < \theta^*$ issue debt.

(ii) *Proof of part 2.* Let

$$\theta^d(\theta^*) = \frac{1}{F(\theta^*)} \int_b^{\theta^*} \theta f(\theta) d\theta$$

and

$$\theta^e(\theta^*) = \frac{1}{(1-F(\theta^*))} \int_{\theta^*}^{1-b} \theta f(\theta) d\theta$$

If $y \leq 1/2$, then for any $\theta^* \in (b, 1-b)$, we have $b < \theta^d(\theta^*) < 1/2$ and:

$$V_d(b, \theta^d(\theta^*)) > 1 - 2b > V_e(b, \theta^e(\theta^*))$$

Similarly, since $\theta^d(\theta^*) < 1/2 < 1-b$ and $\theta^e(\theta^*) < 1-b$

$$V_d(1-b, \theta^d(\theta^*)) < 1 - 2b < V_e(1-b, \theta^e(\theta^*))$$

Now define $\psi(\theta^*) = V_d(\theta^*, \theta^d(\theta^*)) - V_e(\theta^*, \theta^e(\theta^*))$. From the previous results $\psi(b) > 0 > \psi(1-b)$, hence from the intermediate value theorem there exists $\theta^* \in (b, 1-b)$ such that $\psi(\theta^*) = 0$. This proves existence because $V_e(\theta, \theta^e(\theta^*))$ is increasing in θ, while $V_d(\theta, \theta^d(\theta^*))$ is decreasing in θ when $\theta^d(\theta^*) < 1/2$. *End proof.*

Because the first-best share price in the first period is the same for all types, the types with high θ benefit from their informational advantages in the second period (when in actuality they are really "lemons"). Hence, intuitively the separation equilibrium described in Proposition 4 should exist if the payoff in the given debt strategy is decreasing in θ. A sufficient condition for existence of this equilibrium is that $y \leq 1/2$, which means that the economy is not declining on average. Also, in this case, pooling with debt is not an equilibrium. The intuition for this condition is the following: pooling with debt is unprofitable for stagnating firms because, since there are more growing firms, interest rate profile corresponds more with them and, as we know, stagnating firms would lose by playing debt. Thus, stagnating firms tend to signal their type by playing equity. This leads to the following result.

Proposition 5. *1) If and only if $y \geq 1/2$, pooling with d is an equilibrium; 2) if $y < 1/2$, pooling with e is not an equilibrium; 3) if $y \geq 1/2$ and if pooling with e exists, then mispricing is greater under that than under pooling with d.*

Proof: see Appendix 2.

As we can see, the only qualitative difference with the basic model is the fact that here the existence of "separation" equilibrium, outlined in Proposition 4, is subject to the condition that the economy is non-declining. This is not surprising because in a two-type model, like in the one described in Section 3, the grower firm is also growing absolutely, which implies that the interest rate profile is necessarily downward sloping, making it unprofitable for the stagnating firm to mimic this type by playing debt. In the multiple type case, the condition $y \leq 1/2$ insures that in the case of "separation", the equilibrium interest rate profile would necessarily be downward sloping.

5 Conclusion.

Let us summarize the analysis of the model. Two equilibriums may exist: separating and pooling. A separating equilibrium exists if and only if the economy is growing on average. In separating equilibrium firms issuing equity have higher performance in the first period and lower performance in the second period. Also equity may be issued only in the case of separating equilibrium because pooling with debt dominates pooling with equity by minimal mispricing when economy is stagnating. One can see that these results contrast the standard GB approach ("good" and "bad" firms) where the solution is typically pooling with debt. We believe that the idea of signalling the performance profile over time (in contrast or in addition) to the signalling about total performance provides an interesting insight which can be applied in other fields of research as well.

Also, the results of the paper are consistent with several financial phenomena:

(i) Firms issuing equity underperform in the long-run as compared to non-issuing firms (measured as a decline of profit, profit to assets ratio or profit per share). This is implied by Propositions 1 and 4: in any equilibrium, where both debt and equity are issued, only the types with low second period profit issue equity in the first period.[17] At the same time the performance of firms issuing equity exceeds the performance of the non-issuing firms at the time of issue (or in the near future after issue). Clearly, this also follows from Propositions 1 and 4.[18] Similarly, the model predicts that leverage is negatively correlated with profitability. In separating equilibrium, first-period low-profitable firms issue debt.[19]

(ii) This paper suggests a new motive for issuing equity (Propositions 1 and 4) that has not been explored in existing literature. When the firm knows that it will be high-profitable in the near future and low-profitable in the long-term, the entrepreneur may want to issue equity.

(iii) This model provides a rationale for the link between debt-equity choice and business cycle. The analysis of the basic model reveals the following ideas. Growing firms prefer debt. The incentives for stagnating firms depend on the macroeconomic situation. If the economy is in contraction, stagnating firms may prefer debt, but will necessarily issue equity when the economy is growing (Propositions 1, 2, 3 and 4). Thus, equity issues seem to be procyclical.

Acknowledgments. Claude Fluet, Thomas Noe, Leon Petrosjan and P. Viswanath have given us especially useful comments on earlier versions of this paper. We are also grateful to seminar participants at the University of Guelph, ESADE, University of Bonn, UQAM, the Bank of Canada, the 2003 MWF, the 2002 CIRPEE and the 2003 SCSE meetings for their suggestions and comments. The financial support awarded by the Social Sciences and Humanities Research Council of Canada and the Institut de finance mathématique de Montréal was instrumental in enabling our continued research. We also appreciate the editing assistance of Kaarla Sundström.

[17] This conclusion is confirmed by empirical findings (see for example [5], [11], [15] or [16]).

[18] While this point was not the main focus of the empirical research cited above, some authors did stress the point that issuing firms outperform non-issuing firms just before issue, and others documented that issuing firms outperform non-issuing firms in the year of issue and in the first year after issue (see [11] and [16]).

[19] This is consistent, for example, with [9], [20] and [21].

Appendix 1

(i) *Proof of part 1.* We will show that pooling with e does not survive intuitive criterion. Consider pooling with e and assume that if a type deviates to d, it is perceived as the type $\widehat{\theta}_d$. It is enough to show that

$$\min_y V_e(\theta_S, y) \geq \max_{\widehat{\theta}} V_d(\theta_S, \widehat{\theta}_d) \tag{17}$$

where $y < 1/2$ and $\theta_I \leq \widehat{\theta}_d \leq \theta_S$. 1) $\theta_S < 1/2$. From Lemma 3, $V_d(\theta_S, \widehat{\theta}_d) < 1 - 2b$. Since by Lemma 2 $V_e(\theta_S, y) > 1 - 2b$, this proves (17). 2) $\theta_S \geq 1/2$. From (9) minimal value of $V_e(\theta_S, y)$ on feasible support is obtained when $y = 1/2$ (and equals $\frac{1-2b}{1-b}(1 - 2b + 2b\theta_S)$). From (10)

$$V_d(\theta_S, \widehat{\theta}_d) = 1 - b(\frac{\theta_S}{\widehat{\theta}_d} + \frac{1-\theta_S}{1-\widehat{\theta}_d}) \tag{18}$$

Local minimum of expression in brackets under positive $\widehat{\theta}_d$ is $\widehat{\theta}_{d\min} = \frac{\theta_S - \sqrt{\theta_S(1-\theta_S)}}{2\theta_S - 1}$. Thus, the maximal payoff of S, if it were playing d, would be:

$$1 - b(2\theta_S - 1)(\frac{\theta_S}{\theta_S - \sqrt{\theta_S(1-\theta_S)}} + \frac{1-\theta_S}{\sqrt{\theta_S(1-\theta_S)} - 1 + \theta_S})$$

Hence (17) is equivalent to the following:

$$\frac{1-2b}{1-b}(1 - 2b + 2b\theta_S) > 1 - b(2\theta_S - 1)(\frac{\theta_S}{\theta_S - \sqrt{\theta_S(1-\theta_S)}} + \frac{1-\theta_S}{\sqrt{\theta_S(1-\theta_S)} - 1 + \theta_S}) \Leftrightarrow$$

$$(1-b)(2\theta_S - 1)^2\sqrt{\theta_S(1-\theta_S)} - (2\theta_S^2 - 2\theta_S + \sqrt{\theta_S(1-\theta_S)})(3 - 4b + 4b\theta_S - 2\theta_S) > 0$$

Since $\theta_S \geq 1/2$, the left side is decreasing in b. Thus it is enough to show that this inequality holds under $b = 1 - \theta_S$ (because $1 - b \geq \theta_S$). It can be verified that $\theta_S(2\theta_S - 1)^2\sqrt{\theta_S(1-\theta_S)} - (2\theta_S^2 - 2\theta_S + \sqrt{\theta_S(1-\theta_S)})(6\theta_S - 4\theta_S^2 - 1) > 0$ on feasible support of θ_S. Thus, the market would attribute the probability 0 to the possibility of playing d by S. The market would believe that firm is type I after observing debt. In this case, type I would certainly deviate from pooling with e to d.

(ii) *Part 2* follows from Lemma 4. *End proof.*

Appendix 2.

(i) *Proof of part 1.* 1) Sufficiency. Consider pooling equilibrium where all types play d, which is supported by off-equilibrium market beliefs that the firm is the type $1 - b$. First of all, let us verify non-deviation for each type. Consider the case $\theta \geq y$. Since $1/2 \leq y \leq \theta$, we gets from Lemma 3 $V_d(\theta, y) \geq 1 - 2b$. From Lemma 2 we have $V_e(\theta, 1-b) < V_e(\theta, \theta) = 1 - 2b$. Thus the type θ does not deviate. Now, consider the case $\theta < y$. From Lemma 2, we have

$$V_e(\theta, y) \geq V_e(\theta, 1 - b) \tag{19}$$

The condition of non-deviation for the type θ is obviously $V_d(\theta, y) \geq V_e(\theta, 1 - b)$. Given the condition (19), is is enough to show that $V_d(\theta, y) \geq V_e(\theta, y)$. This obviously follows from Lemma 4.

Let us now verify that off-equilibrium beliefs survive the intuitive criterion of Cho and Kreps (1987). To show this, let us calculate the maximal payoff of type $1 - b$ in the case that it plays e. Its payoff is evidently maximized if the market's belief places the probability 1 on type b observing equity, i. e. $V_e(1-b,b)$. If off-equilibrium beliefs survive intuitive criterion, this expression must be greater than $V_d(1-b,y)$. From Lemmas 2 and 4 we get immediately:

$$V_e(1-b,b) \geq V_e(1-b,y) > V_d(1-b,y)$$

This completes the proof of sufficiency.

2) As for the necessary condition, if $y < 1/2$ then pooling with d is impossible because type $1-b$ would deviate in e (its equilibrium payoff would be less than $1-2b$).

(ii) *Proof of part 2.* We will show that pooling with e does not survive intuitive criterion when $y < 1/2$. First, for θ such that $\theta > y$ it can be shown (analogously to Proposition 4) that

$$\min_y V_e(\theta,y) \geq \max_{\widehat{\theta}} V_d(\theta,\widehat{\theta}) \tag{20}$$

if $y < 1/2$. Thus, the market would attribute the probability 0 to the possibility of playing d by θ if $\theta > y$. Evidently in this case $\theta^d < y$. Thus $V_d(\theta^d,\theta^d) = 1-2b$. Also, $V_e(\theta^d,y) < 1-2b$. Thus θ^d would deviate and pooling with e is impossible.

(iii) *Proof of part 3.* Let $y \geq 1/2$. According to part 1, pooling with d exists. Assume that pooling with e exists. From Lemma 2 $V_e(\theta,y) < 1-2b$ for and only for $\theta < y$. As implied by Lemma 4 for such types $V_d(\theta,y) \geq V_e(\theta,y)$. Thus mispricing under pooling with e is larger that under pooling with d. *End proof.*

References

[1] Akerloff G. (1970). The Market for Lemons: Quality Incertainty and the Market Mechanism. Quarterly Review of Economics. Vol. 74 (n. 3), 488-500.

[2] Brennan, M., & Kraus, A. (1987). Efficient Financing under Asymmetric Information. The Journal of Finance, Vol. XLII, n. 5, december, 1225-1243.

[3] Brick, I., Frierman M., & Kim, Y. K. (1998). Asymmetric information Concerning the Variance of Cash Flows: The Capital Structure Choice. International Economic Review, Vol. 39, n. 3 (Aug.), 745-761 .

[4] Cadsby, C.B., Frank, M. & Maksimovic, V. (1998). Equilibrium Dominance in Experimental Financial Markets. The Review of Financial Studies, Vol. 11, No. 1. (Spring), 189-232.

[5] Cai, J., & Wei, K. (1997). The Investment and Operating Performance of Japanese Initial Public Offerings. Pacific-Basin Finance Journal, 5, 389-417.

[6] Cho, I. K., & Kreps, D. (1987). Signalling Games and Stable Equilibria. Quarterly Journal of Economics, 102 (2), 179-221.

[7] Dittmar, A., Mahrt-Smith, J. & Servaes, H. (2003). International Corporate Governance and Corporate Cash Holdings. Journal of Financial and Quantitative Analysis, Vol. 38, n. 1 (march), 111-133.

[8] Easterbrook, F. (1984). Two Agency-cost Explanation of Dividends. American Economic Review, Vol. 74 (4), 650-659.

[9] Fama, E.,& French, K. (2002). Testing Trade-off and Pecking Order Predictions about Dividends and Debt. The Review of Financial Studies, Vol. 15, n.1 ,1-33.

[10] Fudenberg, D. & Tirole, J. (1991). Game Theory. Cambridge, Mass. : MIT Press.

[11] Jain, B., & Kini, O. (1994). The Post-Issue Operating Performance of IPO Firms. The Journal of Finance, vol. XLIX, n.5, December, 1699-1726.

[12] Jensen, M. C. (1986). Agency Cost of Free Cash-flow, Corporate Finance and Takeovers. American Economic Review, Vol. 76, n. 2, 323-329.

[13] Kreps, D. M., & Wilson, R. (1982). Sequential Equilibria. Econometrica, vol. 50, n.4, July, 863-894.

[14] Lang, L., & Litzenberger, R, (1989). Dividend Announcements: Cash Flow Signalling vs. Free Cash Flow Hypothesis? Journal of Financial Economics, 24, 181-191, North-Holland.

[15] Loughran, T., & Ritter, J. (1997). The Operating Performance of Firms Conducting Seasoned Equity Offerings. Journal of Finance, Vol. 52, No. 5, Dec., 1823-1850.

[16] Mickelson, W., Partch, M. & Shah, K. (1997). Ownership and Operating Performance of Companies That Go Public. Journal of Financial Economics, 44, 281-307.

[17] Myers, S.C., & Majluf, N.S. (1984). Corporate Financing and Investment Decisions When Firms Have Information That Investors Do not have. Journal of Financial Economics, 13(2), June, 187-221.

[18] Nachman, D.C., & Noe, T.H. (1994). Optimal Design of Securities Under Asymmetric Information. The Review of Financial Studies, Spring, Vol. 7, No. 1, 1-44.

[19] Petrosian, L. A., & Zenkevich, N. A. (1996). Game Theory. Singapore; River Edge, N. J.; World Scientific, Series on optimization, v. 3.

[20] Rajan, R. G., & Zingales, L. (1995). What Do We Know About Capital Structure? Some evidence from international data. Journal of Finance, 50, 1421-1460.

[21] Titman, S., & Wessels, R. (1988). The Determinants of Capital Structure Choice. Journal of Finance, 43, 1-19.

[22] Zenkevich, N. A., & Voznyuk, S. N. (1996). A Game-Theoretic Model of Divisible Goods Bargaining. Game Theory and Applications. Vol. 1. New York, USA. Nova Science Publishers Inc., 201-208.

[23] Zenkevich, N. A., & Huang, S. (2003). Comparison of Two Economic Models for a Business-to-Business Exchange. ICM Millenium Lectures on Games, Springer-Verlag, Berlin, Heidelberg, 335-345.