Property rights and earnings manipulation*

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Abstract. Existing literature on property rights stresses the effect that distortions in future investment decisions have on establishing the optimal property rights. This paper demonstrates that property rights may also be affected by contracts which exist prior to the establishment of property rights. We consider a two-period model where a firm’s claimholders have contracts on current earnings and must determine the allocation of property rights on the firm’s residual assets. The allocation of these rights affects the claimholders’ incentives to undertake optimal financial decisions which simultaneously affects current cash flows and the firm’s residual value. We argue that property rights should be connected to the existing contracts through the rule of marginal revenues in order to mitigate the intertemporal substitution (earnings manipulation) problem.

Key words: earnings manipulations, intertemporal substitution, design of securities, property rights, cash flow rights, control rights

JEL classification codes: G32, D92, D82

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1 Introduction.

The rules governing the allocation of property rights have become a major topic in economic and financial theory. It is difficult to optimally distribute a large number of various rights between shareholders, creditors, managers, etc. The best known contribution in this area is the "property rights approach" (PRA). This approach is based on the observation that certain types of arrangements may be contractible (non-residual actions and rights) while others are not contractible. In the latter case, it is impossible to write complete contracts which determine the optimal actions in any scenario (residual actions and rights). Therefore, control over residual actions becomes important. The allocation of property rights, which are usually contractible, determines which party has control. Subsequent PRA related literature has presented several theories concerning firms’ financial structures which are based on the allocation of control rights. Early examples of such analysis predict that a combination of debt and equity is optimal. Such financial structures allow an efficient distribution of control rights. These typically use the following rule: if a firm performs well, control goes to the shareholders, and control passes to the creditors otherwise.

Empirical literature in this field generally confirms the fact that the shareholders retain control when the business is performing relatively well while control passes to the creditors when the opposite is true. This literature also stresses that, while in real financial contracts the allocation of control rights plays an important role, the parties involved usually make separate determinations for the allocation of various kinds of property rights. These rights include residual cash flow rights, liquidation rights, board rights etc. Moreover, there are significant quantitative differences between the percentages of different rights allotted to the parties. Recent literature on financial contracts involving the allocation of property rights has made two important contributions. First, it has explained why, in many cases (especially within venture capital firms), investors usually hold convertible securities and not

3See, for instance, Kaplan and Stromberg (2003) and Elfenbein and Lerner (2003) with regard to venture capital contracts and Gilson (1990) for corporate contracts.
simply debt. Second, it has explained why the difference between control rights and property rights may exist. However, many questions regarding the links between security holders’ different rights remain unanswered. For instance, Kaplan and Stromberg (2003, p. 313) write: "Our results clearly show that real world contracts are more complex than existing theories predict. For example..., the allocations of cash flow and control rights and the use of contingencies are related in systematic ways..."

This paper points out that property rights may also be affected by contracts which exist prior to the establishment of property rights. We consider a situation where two parties bargain about the property rights on a firm’s residual assets. We assume that the resulting fractions of ownership entitle each party to the right to the corresponding fraction of the firm’s future earnings (the results of the paper will still hold if there is deviation from the one share-one vote rule as long as the parties rationally anticipate this difference). In contrast to most existing literature, the parties have issued contracts on the firm’s current earnings (hereafter NR) prior to determining the allocation of property rights. Essentially, the paper analyzes the links between these contracts and property rights (RR). We look at this link through the intertemporal substitution (earnings manipulation) argument.

Earnings manipulation is different from earnings misreporting. The latter means presenting incorrect information about a firm’s results to the claimholders and the public. The central characteristic of the former is intertemporal substitution: the decision-maker may engage in activities which move cash flows from one period to another. Some typical examples include delaying the approval of important decisions, inefficient investments, borrowing in order to manipulate financial results, inefficient discount policy etc. The firm can also use different accounting strategies such as the choice of inventory and depreciation methods, the expensing of research and development, the estimation of pension liabilities, and the capitalization of leases and marketing expenses. When an action from the above list has a positive social value, earnings management takes place, otherwise - and this is the focus of the present paper - earnings manipulation takes place. If the decision-maker is the firm’s manager, the objective of earnings manipulation


\footnote{Kirilenko, 2001.}

\footnote{Roychowdhury (2006) provides extensive evidence on earnings management through real activities manipulation.}
is usually to increase the firm’s short-term performance and thereby achieve some threshold. The reasons for managers’ "short-termism" are well known. The most important reason is the difference between a firm’s life horizon and a manager’s expected period of employment. However, if the firm’s major claimholders are involved in intertemporal substitution, the target is not only the current payoffs to the holders of securities but also (and perhaps first of all), the residual rights of the holders of securities and, most importantly, property rights.

Teoh, Welch and Wong (1998) found that IPOs (initial public offerings), which are typically the major event affecting the distribution of property rights, are almost always preceded by aggressive earnings management. In their conclusion, the authors stress that investors must carefully interpret accounting information provided by the issuing firm. In our view, this can only be done by understanding the link between the allocation of property rights in the new firm and the incentive to manipulate earnings. Our analysis shows that the allocation of property rights may significantly affect the claimholders’ incentive to engage in intertemporal substitution. For instance, if the decision-maker is the only owner of the future business, he will be interested in shifting value from the current firm to the future firm even if this action is socially inefficient.

We argue that the optimal situation occurs when RR are distributed according to the current marginal revenues of existing securities. For instance, if the current marginal cash flow rights of the decision-maker are 15% (if the firm’s value increases by $1, the payoff to the agent will increase by 15c), his optimal fraction of ownership in the future business should also be 15%. This provides decision-makers with the optimal incentives to choose intertemporal substitutions.

The problem with this simple argument is that, since current cash flow rights are usually non-linear, a large manipulation of earnings (going beyond the threshold) can lead to disproportionate changes in marginal current cash flow and RR. We show, however, that the marginal revenue rule may work. **Optimal incentives can be implemented when** contracts are piece-wise linear, the parties observe certain signals regarding the state of nature, and the allocation of control rights is state-contingent (optimal incentives can be implemented - delete). (Sorry - I didn’t read it as a list - numeration is not necessary - rearranging the sentence clarifies) As is typically analyzed in the literature, standard securities such as debt, equity, and convertible equity are consistent with this theory. An interesting direc-
tion for further empirical research would be to verify this rule’s consistency when applied to other securities.\footnote{This would be interesting because the set of observable securities is much larger than just the set of standard securities. For example, Brealey, Myers and Allen (2005, p. 964) write: “In the last 20 years companies and security exchanges have created an enormous number of new securities: options, futures, options on futures, zero-coupon bonds […] the list is endless.”}

The recent wave of scandals in the corporate world (Worldcom, Enron, Nortel etc.) has shown that earnings manipulation is playing an important role in corporate life. Existing theoretical literature typically focuses on the implications of earnings manipulation on the design of managers compensation contracts (see, for instance, Degeorge, Patel and Zeckhauser (1999), Jensen (2003) and Crocker and Slemrod (2005)). In Degeorge, Patel, and Zeckhauser (1999), a manager with a bonus-like contract may manipulate earnings in order to exceed some threshold. In the present paper, we focus on how earnings manipulation affects the allocation of different rights between security holders. We look at the problem not as a single manager moral hazard problem, but rather as a problem where the decision-making process is split among different security holders. Under this approach, the paper’s results can be applied to entrepreneurial firms, venture capital firms, corporations with a large shareholder, and any other firms where investors are involved in the decision-making process.

The rest of the paper is organized as follows: Section 2 describes the model; Sections 3 and 4 analyze optimal property rights when current cash flow rights are two-part linear and three-part linear respectively. Section 5 discusses the model’s results with regard to empirical evidence and Section 6 presents the conclusion.

\section{Model.}

Consider a firm with assets in place which are expected to generate a cash flow $R$ in the first period. The cash flow is distributed according to the distribution function $F(R)$ and the density function $f(R)$; $R \in [0, \bar{R}]$. The company’s assets which remain at the end of first period may yield the revenue $C$ in the second period. The firm’s final first-period profit is $R_1 = R - a$, where $a$ is a profit correction arising from intertemporal substitution.
Earnings in period 2 are \( R_2 = C + c(a), \ c' \geq 0, \ c'' \leq 0, \ c(0) = 0 \). \(^8\)

Before \( R \) is realized, the parties should establish property rights, \( \alpha_j(R_1) \), on the assets which remain at the end of first period (accordingly \( \alpha_j(R_1)R_2 \) is the second-period payoff of claimholder \( j; j \in \{1, 2\} \)). We have

\[
\alpha_1(R_1) + \alpha_2(R_1) = 1, \forall R_1
\]

where

\[
0 \leq \alpha_j(R_1) \leq 1, \forall j
\]

The firm has issued two claims: for claimholders \( A \) and \( B \) respectively. \( A\)’s contract (his first-period payoff) is \( T_1(R_1) \) and that of \( B \) is \( T_2(R_1) \).

\[
T_1(R_1) + T_2(R_1) = R_1, \forall R_1
\]

Let us assume:

(i) \( T_j(R_1), \forall j \) are monotonic and either two-part or three-part linear functions from \([0, R]\) into \([0, R]\). Denote the breaking point of two-part contracts by \( R_{t1} \) and the breaking points of three-part contracts by \( R_{t1} \) and \( R_{t2} \), where \( R_{t1} < R_{t2} \). \(^9\) \( T_j(R_1), j \in \{1, 2\} \) cannot be renegotiated. \(^10\)

(ii) Earnings are non-negative in both periods, i. e. \( R_1 \in [0, R], a \in [R - R, R] \) and \( C \in \mathbb{C}, -\infty \), where \( \mathbb{C} = -c(-R) \). Since \( c(a) \) is increasing in \( a \), this condition guarantees that

\[
R_2 = C + c(a) \geq 0
\]

(iii) \( a^* = \arg \max_a [c(a) - a] = 0. \)

It follows from the model description that: 1) intertemporal substitution activities affect current profit and residual benefits in opposite directions; 2) \( a = 0 \) is socially optimal because \( a^* = \arg \max_a (R - a + C + c(a)) \), where the expression in brackets represents the firm’s total earnings; 3) the negative

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\(^8\)In this setting an "investment" can be seen as an action with \( a > 0 \) and a liquidation of assets or borrowing can be seen as an action with \( a < 0. \)

\(^9\)Only piece-wise linear contracts are considered since these are the most common type of contracts observed in reality. The case of linear contracts is not considered because the problem of intertemporal substitution does not arise (see Jensen, 2003). Also, contracts with a high-number-of-pieces (more than 3) are not considered to be realistic. However, the analysis can easily be extended to those cases.

\(^10\)This condition insures that the Coase theorem (1960), which concerns the irrelevance of property rights, does not apply.
The impact of intertemporal substitution activities increases with deviations from $a^*$ and; 4) the condition (2) and assumptions (i) and (ii) imply two-sided limited liability in both the first and second periods.\footnote{See Innes (1990) and Robe (1999) for theoretical and practical issues related to limited liability rules.}

$A$ and $B$ meet to discuss the optimal choice of $a$. We assume that both parties are not wealth-constrained. If $R$ is verifiable, they can write a contract which imposes large sanctions on each party if $R_1 \neq R$ (both parts of this condition are verifiable by a court). If the penalties are large enough, the decision-maker will choose $a = a^*$. The same holds if $a$ is verifiable. Also, if the parties have veto power on any transaction then the problem of intertemporal substitution will disappear: from assumption (iii) any $a \neq 0$ reduces the payoff of at least one party. However, such a monitoring or decision-making process is costly and unrealistic. Hence, we assume that only $R_1$ is verifiable. Therefore, we focus on the following situation. $A$ is the initial decision-maker, i. e. he has the right to choose $a$. After the parties observe $R$, they may renegotiate the control rights.\footnote{The usage of different kinds of signaling is typical in financial contract literature. This is based on Kaplan and Stromberg (2003) in that the allocation of different rights is usually contingent on some measures of intermediate performance. For other kinds of signaling see Zender (1991), Aghion and Bolton (1992), Dewatripont and Tirole (1994) and Hellman (2005).} This means that $A$ can sell the control rights to $B$. Following this, the party which has control chooses $a$.

As in the property rights approach, it is assumed that, since the parties are able to make payments to each other, they will choose the property rights which maximize the total surplus. Property rights should provide the decision-maker with the optimal incentive to choose $a$. The game is as follows:

\begin{enumerate}
  \item Property rights $\alpha_j(R_1), j \in \{1, 2\}$ are established. A money transfer is made if necessary.
  \item The parties observe $R$, and $A$ may sell the control rights.\footnote{Since neither party is wealth-constrained, the specific mechanics of bargaining about the price in stages 1 and 2 is irrelevant. It can affect the price but not the optimal features of security design. The optimal scenario, which maximizes the total surplus, will always be chosen.} Define $d(R)$ (control) as follows: $d(R) = 1$ if $A$ retains control and $d(R) = 2$ if $A$ sells control to $B$.
  \item If $d(R) = 1$, $A$ chooses $a$. If $d(R) = 2$, $B$ chooses $a$.
  \item $R_1$ becomes known ($R_1 = R - a$) and the parties receive their first-}

...
period payoffs $T_j(R_1)$. $\alpha_j(R_1), j \in \{1, 2\}$ become known.

5. $R_2$ is realized ($R_2 = C + c(a)$) and the parties receive their residual payoffs $\alpha_j(R_1)R_2$.

3 Two-part linear contracts.

First, let us consider the incentive of an agent $j$ with a two-part linear contract $T_j$ assuming that the residual (second-period) property rights equal the current marginal cash flow rights: $\alpha_j(R_1) = T_j'(R_1)$. Let

$$
\begin{cases}
  T_j = k_{j1} R_1, & R_1 \leq R_{t1} \\
  T_j = k_{j1} R_1 + k_{j2} (R_1 - R_{t1}), & R_1 > R_{t1}
\end{cases}
$$

(5)

$$
\begin{cases}
  \alpha_j = k_{j1}, & R_1 < R_{t1} \\
  \alpha_j = k_{j2}, & R_1 > R_{t1} \\
  \alpha_j = \max \{k_{j1}, k_{j2}\}, & R_1 = R_{t1}
\end{cases}
$$

(6)

where $k_{j1} \neq k_{j2}$. For our purposes, the following analysis will be useful. Let $W_j(a, R)$ be the final payoff of claimholder $j$ if $R$ is realized and action $a$ is undertaken. Let $a_j^* = \arg \max_a W_j(R, a)$. If $a_j^* = 0$ and $j$ is the decision-maker, the decision is optimal. By definition

$$
W_j(R, a) = T_j(R - a) + \alpha_j(R - a)(C + c(a))
$$

Given (5)-(6), this is equivalent to:

$$
W_j(R, a) = 
\begin{cases}
  k_{j1}(R - a) + k_{j1}(C + c(a)), & R - a < R_{t1} \\
  k_{j1}R_{t1} + k_{j3}(C + c(R - R_{t1})), & R - a = R_{t1} \\
  k_{j1}R_{t1} + k_{j2}(R - a - R_{t1}) + k_{j2}(C + c(a)), & R - a > R_{t1}
\end{cases}
$$

(7)

The following is useful.

**Lemma 1.** $W_j(R, a)$ is discontinuous in $a$ when $a = R - R_{t1}$ and is continuous otherwise. Furthermore

$$
\frac{\partial W_j(R, a)}{\partial a} \geq 0 \text{ if } a < 0 \text{ and } R_{t1} \neq R - a
$$

(8)

$$
\lim_{a \to R - R_{t1} + 0} W_j(R, a) \geq \lim_{a \to R - R_{t1} - 0} W_j(R, a) \text{ if } k_{j1} > k_{j2}
$$

(9)
Proof. The first part follows directly from (7) and \( k_{j1} \neq k_{j2} \). Next from (7), \( \frac{\partial W_j(R,a)}{\partial a} = T'_j(R - a)(\frac{\partial c(a)}{\partial a} - 1) \). From assumption (i) \( T'_j(R - a) \geq 0 \). Therefore, (8) follows from assumption (iii) and \( c'' \leq 0 \). The condition (9) follows from

\[
\lim_{a \to R - R_{t1}} W_j(R, a) - \lim_{a \to R - R_{t1}} W_j(R, a) = (k_{j1} - k_{j2})(C + c(R - R_{t1}))
\]

and assumption (ii). End proof.

Intuitively, the payoff function is discontinuous at the break-points because this leads to a jump in RR. The condition (8) is based on the idea that increasing the magnitude of profit correction is unprofitable when RR do not change.

Let

\[
s = \begin{cases} 
1 & \text{if } R < R_{t1} \\
2 & \text{if } R > R_{t1}
\end{cases}
\]

Figure 1 illustrates Lemma 1 for the case of a concave contract.

**Figure 1.** \( W_j(R, a) \) when \( k_{j1} > k_{j2} \) and: a) \( s = 1 \); b) \( s = 2 \).

When \( R_{j1} > R - a \) (Figure 1a) the optimal choice is \( a = 0 \) because no deviation from this can increase RR. In contrast, if \( R_{j1} < R - a \), such deviations can be profitable because they may increase RR. As one can see, two possibilities exist. If the benefits from increasing RR are more important than the loss in social profit then a deviation toward the break-point
is optimal (thick line on the right side of Figure 1b). If not, the optimal $a = a^*$ (doted line). Also, in both cases it is never profitable for the concave claimholder to have $a < a^*$.

**Lemma 2.** 1) Let $k_{j1} > k_{j2}$. If $s = 1$ then $a^*_j = 0$; 2) Let $k_{j1} < k_{j2}$. If $s = 2$ then $a^*_j = 0$.

**Proof.** Let $k_{j1} > k_{j2}$ and consider the case where $R < R_{t1}$. From Lemma 1,

$$W_j(R, a | a < R-R_{t1}) \leq \lim_{a \to R-R_{t1}-0} W_j(R, a) \leq \lim_{a \to R-R_{t1}+0} W_j(R, a) \leq W_j(R, 0)$$

and

$$W_j(R, 0) \geq W_j(R, a | a \geq R - R_{t1})$$

(see also Figure 1a). Thus, $a^*_j = 0$. The argument is analogous for the case $k_{j1} < k_{j2}$ and $R > R_{t1}$. End proof.

It is clear from Lemma 2 that when $R < R_{t1}$, it is optimal for the concave claimholder to have control. If, in contrast, $R > R_{t1}$, it is optimal for the convex claimholder to have control.

**Lemma 3.** 1) Let $k_{j1} > k_{j2}$ and $s = 2$. If $| R - R_{t1} |$ is sufficiently small then $a^*_j \neq 0$; 2) Let $k_{j1} < k_{j2}$ and $s = 1$. If $| R - R_{t1} |$ is sufficiently small then $a^*_j \neq 0$.

**Proof.** First consider the case where $k_{j1} > k_{j2}$ and $s = 2$. Let us compare $W_j(R, 0)$ and $\lim_{a \to R-R_{t1}+0} W_j(R, R_{t1} - R)$. From (7) the difference between them equals:

$$\lim_{R \to R_{t1}+0} [k_{j1}R_{t1} + k_{j2}(R - R_{t1}) + k_{j2}C - k_{j1}R_{t1} - k_{j1}(C + c(R_{t1} - R))] =$$

$$(k_{j2} - k_{j1})C < 0$$

Since the difference between $W_j(R, 0)$ and $\lim_{R \to R_{t1}+0} W_j(R, R_{t1} - R)$ is strictly negative, the lemma follows from the continuity of $W_j(R, R_{t1} - R)$ for $R > R_{t1}$ which is implied by Lemma 1. The argument is analogous for the case where $k_{j1} < k_{j2}$ and $s = 1$. End proof.

The meaning of Lemma 3 is that if $k_{j1} > k_{j2}$ and $s = 2$ or $k_{j1} < k_{j2}$ and $s = 1$, the decision-maker’s choice cannot be optimal for all $R$. This

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14We assume that if $j$ is indifferent between $a^*$ and any other $a$, he will choose $a^*$. 

10
makes the transfer of control rights necessary. For instance, the entrepreneur (typically the claimholder with a convex contract or with a contract where \( k_{j1} < k_{j2} \)) is often reluctant to bankrupt the firm even if it is socially optimal \((s = 1)\). This is because it usually leads to low residual rights for the entrepreneur.\(^{15}\)

Now we can formulate the conditions under which the optimal \( a \) is chosen.

**Proposition 1.** Let \( T_1(R_1) \) be two-part linear. The optimal \( a \) \((a = 0)\) is chosen for any \( R \) if and only if: 1) \( \alpha_j(R_1) = T_j'(R_1), \forall R_1 \neq R_{t1}, \forall j \in \{1, 2\} \) and/or? 2) \( T_1(R_1) \) is convex and \( d(R \mid s = 1) = 2, \ d(R \mid s = 2) = 1, \) or \( T_1(R_1) \) is concave and \( d(R \mid s = 1) = 1, \ d(R \mid s = 2) = 2. \)

**Proof.** Part 1. Sufficiency. Let \( \alpha_1(R_1) = T_1'(R_1), \forall R_1 \neq R_{t1} \) and \( \alpha_1(R_{t1}) = \max\{k_{11}, k_{12}\}. \) Obviously \( T_1(R_1) \) satisfies (5) and (6). The conditions (3), (1), and (2) imply that \( T_2(R_1) \) and \( \alpha_2(R_1) \) also satisfy (5) and (6) because \( \alpha_2(R_{t1}) = 1 - \max\{k_{11}, k_{12}\} = \min\{1 - k_{11}, 1 - k_{12}\} \leq \max\{1 - k_{11}, 1 - k_{12}\}. \) Therefore, lemmas 1 and 2 can be applied for both agents. Thus, sufficiency follows from Lemma 2. More precisely, consider the case when \( A \)'s contract is convex (it can be shown analogously for the concave case). If \( s = 2 \) then \( A \) retains control. According to Lemma 2, \( A \)'s choice will be optimal. If \( s = 1 \) then \( B \) gets control and his decision will be optimal.

Necessity. Consider the incentive of \( j \) regarding the choice of \( a \) when \( R \neq R_{t1} \). Note that such a state of nature has a positive Borel measure. We have:

\[
W_j(R, a) = T_j(R - a) + \alpha_j(R - a)(C + c(a))
\]

\[
\frac{\partial W_j}{\partial a} = -T_j'(R_1) - \alpha_j'(R_1)(C + c(a)) + \alpha_j(R_1) \frac{\partial c(a)}{\partial a}
\]

For \( a = 0 \) we have:

\[
\frac{\partial W_j}{\partial a} \bigg|_{a=0} = -T_j'(R) - \alpha_j'(R)C + \alpha_j(R) \frac{\partial c(a)}{\partial a} \bigg|_{a=0}
\]

According to assumption (ii), \( \frac{\partial c(a)}{\partial a} = 1 \) when \( a = 0 \). Thus, \( \frac{\partial W_j}{\partial a} = -T_j'(R) - \alpha_j'(R)C + \alpha_j(R). \) If this does not equal 0, the choice \( a = 0 \) cannot be optimal for \( j \). Thus, we have the following (differential) equation:

\[
y = x - Cx'
\]

\(^{15}\)See, for instance, Titman (1984).
where $x = \alpha_j(R)$ and $y = T_j'(R)$. The solution of this differential equation is $x = y$ (note that $y' = 0$ because contracts are piece-wise linear and $R \neq R_{t1}$). Hence, we have: $\alpha_j'(R) = 0$ and $\alpha_j(R) = T_j'(R)$. This implies that

$$\alpha_j(R_1) = T_j'(R_1), \forall j \quad (11)$$

given $T_1'(R_1) + T_2'(R_1) = 1, \forall R_1 \neq R_{t1}$.

Part 2. From Part 1, the optimal choice of $a$ cannot be guaranteed if (11) does not hold. Consider the case when it holds and suppose that $A$’s contract is convex (it can be shown analogously for the concave case). If the proposition does not hold, the three alternative possibilities are: $d(R \mid s = 1) = d(R \mid s = 2) = 1$; $d(R \mid s = 1) = d(R \mid s = 2) = 2$ and $d(R \mid s = 1) = 1$, $d(R \mid s = 2) = 2$. None of these can guarantee the optimal choice of $a$ as implied by Lemma 3. End proof.

The condition (10) has an easy interpretation. $T_j'(R_1)$ shows the marginal first-period revenue of $j$ while $\alpha_j(R) - \alpha_j'(R)C$ shows his marginal second-period revenue which includes RR and a marginal correction of RR (in the opposite direction to the changes in first-period revenue). If the former is larger than the latter, $j$ is interested in shifting first-period cash flow to the second period. If the latter is larger than the former, $j$ will inflate first-period profit. The optimal $a$ can only be chosen if (10) holds.

Table 1. Different rights of security holders when contracts are two-part linear.

<table>
<thead>
<tr>
<th>Claim 1</th>
<th>$R &lt; R_{t1}$</th>
<th>$R &gt; R_{t1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Current cash flow rights</td>
<td>convex</td>
<td></td>
</tr>
<tr>
<td>Current control rights</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Residual property rights</td>
<td>Lower than when $R &gt; R_{t1}$</td>
<td>Higher than when $R &lt; R_{t1}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Claim 2</th>
<th>$R &lt; R_{t1}$</th>
<th>$R &gt; R_{t1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Current cash flow rights</td>
<td>concave</td>
<td></td>
</tr>
<tr>
<td>Current control rights</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>Residual property rights</td>
<td>Higher than when $R &gt; R_{t1}$</td>
<td>Lower than when $R &lt; R_{t1}$</td>
</tr>
</tbody>
</table>

Table 1 summarizes the results of the analysis in this section. Note that it can easily be generalized by allowing any number of two-part linear convex
and concave contracts with a break-point \( R_{t1} \). This is because the distribution of current decision rights among convex claimholders (when \( s = 2 \)) or among concave claimholders (when \( s = 1 \)) does not matter: in both cases the decision will be unanimous. This is consistent with the free-rider phenomenon where most claimholders do not really participate in the decision-making process. However, the distribution of property rights should follow the strict rule of marginal revenue for every voting contract.

4 Three-part linear contracts.

The model presented in the previous section illustrates why the parties may suffer from earnings manipulation when the property rights do not follow the rule of marginal revenues (for the case when existing contracts are two-part linear). For the case of three-part linear contracts, there is a set of contracts for which the problem of earnings manipulation can be eliminated through the rule of marginal revenues. However, there also exists a set of contracts for which earnings manipulation is unavoidable. Below we present an outline of the main results (formal derivations are omitted for brevity and are available upon request). Let

\[
\begin{align*}
T_j &= k_{j1}R_1, R_1 \leq R_{t1} \\
T_j &= k_{j1}R_{t1} + k_{j2}(R_1 - R_{t1}), R_{t1} < R_1 \leq R_{t2} \\
T_j &= k_{j1}R_{t1} + k_{j2}(R_{t2} - R_{t1}) + k_{j3}(R_1 - R_{t2}), R_{t2} < R_1
\end{align*}
\]

where \( k_{j1} \neq k_{j2} \) and \( k_{j2} \neq k_{j3} \) and suppose that property rights follow the rule of marginal revenues:

\[
\begin{align*}
\alpha_j &= k_{j1}, R_1 < R_{t1} \\
\alpha_j &= k_{j2}, R_{t1} < R_1 \leq R_{t2} \\
\alpha_j &= k_{j3}, R_{t2} < R_1 \\
\alpha_j &= k_{j4} \leq \max\{k_{j1}, k_{j2}\}, R_1 = R_{t1} \\
\alpha_j &= k_{j5} \leq \max\{k_{j2}, k_{j3}\}, R_1 = R_{t2}
\end{align*}
\]

Consider the case \( k_{11} < k_{12} \) and \( k_{13} < k_{12} \) (Figure 3a, thick line). When \( R_{t1} < R < R_{t2} \), the optimal choice is \( a = 0 \) because no deviation from this can increase RR. In contrast, if \( R < R_{t1} \) or \( R > R_{t2} \), such deviations can be profitable because they may increase RR. Two possibilities exist. If the benefits from increasing RR are more important than the loss in the firm’s total profit then deviating toward the break-point is optimal. This happens
when \( R \) is sufficiently close to the break-point. Thus, if the decision-maker has a contract \( k_{11} < k_{12} \) and \( k_{13} < k_{12} \), the optimal action will be chosen if \( s = 2 \). Otherwise, he may manipulate earnings. We now turn to the second claimholder.

![Figure 3](image_url)

**Figure 3.** a) \( k_{11} < k_{12}, k_{13} < k_{12} \) and \( k_{21} > k_{22}, k_{23} > k_{22} \); b) \( k_{11} > k_{12} > k_{13} \) and \( k_{21} < k_{22} < k_{23} \).

His contract is \( k_{22} < k_{21} < k_{23} \). He will not substitute earnings if \( s = 3 \) (his RR are maximal in this case). Consider the case \( s = 1 \). We show that there exists a set of contracts such that \( a_j^* = 0 \) for any \( R \).

We also show that earnings manipulation is unavoidable under some types of contracts. Consider a contract \( k_{11} > k_{12} > k_{13} \) (Figure 3b, dotted line) and suppose \( s = 2 \). If \( |R - R_{t1}| \) is sufficiently small, \( a_j^* \neq 0 \). The argument here is similar to that in Lemma 3 for two-part linear contracts. If \( R \) is close to the break-point, the decision-maker will manipulate earnings to attain this threshold and increase his RR. The same holds for the second claimholder with a contract \( k_{21} < k_{22} < k_{23} \) when \( s = 2 \). If \( |R - R_{t2}| \) is sufficiently small, \( a_j^* \neq 0 \). Thus, if \( k_{11} > k_{12} > k_{13} \) (or \( k_{11} < k_{12} < k_{13} \)) the first-best social surplus is not achievable.

Table 2 summarizes the optimal design of property rights when the contracts are three-part linear (for the case when the first-best is achievable). As in the previous section, the analysis in this section can be generalized by allowing a large number of claimholders.
Table 2. Different rights of security holders when contracts are three-part linear.

<table>
<thead>
<tr>
<th>Claim 1</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>R &lt; R_{t1}</td>
<td>R_{t1} &lt; R &lt; R_{t2}</td>
<td>R &gt; R_{t2}</td>
<td></td>
</tr>
<tr>
<td>Current cash flow rights</td>
<td>convex for low $R$, concave for large $R$ (Figure 3a, thick line)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Current control rights</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>Residual property rights</td>
<td>Lower than when $R_{t1} &lt; R &lt; R_{t2}$</td>
<td>Higher than when $R &lt; R_{t1}$ or $R &gt; R_{t2}$</td>
<td>Lower than when $R_{t1} &lt; R &lt; R_{t2}$</td>
</tr>
<tr>
<td>Claim 2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Current cash flow rights</td>
<td>concave for low $R$, convex for large $R$ (Figure 3a, doted line)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Current control rights</td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Residual property rights</td>
<td>Higher than when $R_{t1} &lt; R &lt; R_{t2}$</td>
<td>Lower than when $R &lt; R_{t1}$ or $R &gt; R_{t2}$</td>
<td>Higher than when $R_{t1} &lt; R &lt; R_{t2}$</td>
</tr>
</tbody>
</table>

5 Implications.

The interpretation of the model’s results and their contribution to existing literature is as follows.

1. The main result is the "Rule of marginal revenues". The paper suggests a rule for the link between current cash-flow rights and RR. If the current-period’s payoﬀ is $T(R_1)$ then RR should be $T’(R_1)$. This result has not yet been explicitly tested in the literature though it seems to be consistent with standard securities. Suppose, for example, that $B$’s contract is debt and $A$’s contract is levered equity: $T_2(R_1) = \min\{R_1, R_{t1}\}$ and $T_1(R_1) = \max\{0, R_1 - R_{t1}\}$. If cash ﬂow is greater than the face value of debt, $R_{t1}$, then RR belong to the equityholder ($T’_1(R_1) = 1$), and if cash ﬂow is less than the face value of debt, the creditors have all RR ($T’_2(R_1) = 1$). This is consistent with what is observed in practice and with early theoretical literature on this topic (as was mentioned in the introduction). Now consider securities which have been observed more frequently in practice with respect to venture capital firms. Take, for instance, convertible preferred participating equity. The venture capitalist’s (VC’s) cash ﬂow rights correspond to the following two-part linear contract in our model: $T_1(R_1) = \min\{R_1, \beta + \gamma R_1\}$, where $\beta > 0$ and $0 < \gamma < 1$. When the ﬁrm is going public, the VC’s fraction of ownership will be $\gamma$. This usually happens when the business is doing well. If the ﬁrm is doing poorly, the VC’s fraction of ownership is unity. This is
exactly what our model predicts. The argument is analogous for convertible preferred equity: \( T_1(R_1) = \max\{0, R_1 - R_{t1}\} \) if \( R_1 < D \) and \( = D + \gamma(R_1 - D) \) if \( R_1 > D \).

2. The results regarding the allocation of control rights are similar to those found in existing literature with standard securities such as a combination of straight debt and common equity. Our new contributions are as follows. First, we jointly obtain the allocation of control rights and residual property rights. These rights are identical for standard securities (as we argue below) but not for most of other securities. The difference between the allocation of control rights and residual property rights cannot be found in existing literature, for instance, in Aghion and Bolton (1992). Second, we obtain predictions for the allocation of control under three-part linear contracts.

With regard to two-part linear contracts, Proposition 1 predicts that, if interim cash flow is low, control belongs to the investor with a concave claim. Otherwise, control belongs to the investor with a convex claim. To give a classical example: if \( B \)'s contract is debt with face value \( R_{t1} \) and the current-period's profit is below \( R_{t1} \), control should belong to \( B \). Control should belong to \( A \) otherwise. This result holds for any convex or concave contract (not just debt and equity). In particular, the results are consistent with convertible participating preferred equity.

Also, a difference may exist between control rights and RR. For standard debt and levered common equity they are identical: on the upside (when \( R > R_{t1} \)) equity has 100% control and 100% RR, while on the downside (when \( R < R_{t1} \)) debt has 100% control and 100% RR. However, the situation is different for other two part linear contracts where joint ownership is possible on both sides while control is allocated to only one party. Suppose that \( A \)'s contract has the slope 0 on the downside and \( \varepsilon \) on the upside (where \( \varepsilon > 0 \)). It is a convex contract and therefore, when \( s = 1 \), \( A \) is the decision-maker. However, his RR will be small in this case if \( \varepsilon \) is sufficiently low. One can have the same situation with a concave contract.

With regard to three-part linear contracts, the first-best social surplus can only be achieved if the contracts are like those in Figure 3a. In this case, control belongs to \( A \) (thick line) if \( s = 1 \) or \( s = 3 \) and belongs to \( B \) if \( s = 2 \). The natural interpretation of these contracts, for venture capital firms, is that \( A \) is the investor (venture capitalist with convertible preferred equity) and \( B \) is the entrepreneur. The results related to the allocation of control rights for the cases where \( s = 1 \) (investor’s control) and \( s = 2 \) (entrepreneur’s control)
are similar to the cases with two-part linear contracts. The model also predicts that if $s = 3$ control should belong to the investor. This contributes to existing literature which typically fails to explain why the investor often has more control when the firm’s performance is "extremely good" than when performance is simply "good" (see, for instance, Hart, 2001).

3. The situation with three-part linear contracts, where one contract has increasing slopes and another contract has decreasing slopes, never provides the first-best social surplus. Hence, we should not frequently observe contracts like these in practice. To the best of our knowledge this is the case.

6 Conclusion.

Existing literature on property rights stresses the effect that distortions in future investment decisions have on establishing the optimal property rights. This paper demonstrates that property rights may also be affected by contracts which exist prior to the establishment of property rights. It is shown that the rule of marginal revenues is necessary to eliminate intertemporal substitution (earnings manipulation). This result has not yet been explicitly tested in existing literature though it seems to be consistent with standard securities such as debt, equity, and convertible preferred equity. In contrast to most existing theoretical literature, this paper explains why control rights and residual property rights are not usually identical.

References


