R&D-based Economic Growth in a Supermultiplier Model

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Abstract:

We investigate how economic growth in a demand-driven economy with semi-endogenous productivity growth can be compatible with a stable employment path. Our model uses a Sraffian supermultiplier (SSM), and we endogenize the growth rate of autonomous demand, and semi-endogenize productivity growth. The basic model has a steady state that is consistent with a stable employment rate, and in which the growth rate is determined by R&D expenditures. Consumption smoothing (between periods of high and low employment) by workers is the mechanism that ensures that demand keeps up with productivity growth and that the growing economy is stable. We also introduce a version of the model where the burden for stabilization falls upon government fiscal policy. This also yields a stable growth path, although the parameter restrictions for stability are more demanding in this case.

Keywords: Economic growth model, Sraffian supermultiplier, Research and Development (R&D)

JEL Codes: O41, E11, E12, E62

1. Introduction

The core of Keynesian economics is the rejection of Say's Law, or, phrased in a more positive way, the importance of the demand side of the economy. Demand is often broken down into two parts, one part dependent on current income, and another part being autonomous, i.e., independent of current income. This leads to the idea of the multiplier, which, put simply, says that the income-dependent part of demand has important indirect effects, i.e., production that is undertaken to meet (autonomous) demand leads to income (e.g., for workers), which leads to more demand, and, in turn, to more income, etc. If the marginal propensity to consume out of current income is smaller than one, this process will converge (each new round of induced spending is smaller than the previous one) and the level of economic activity (GDP) is found by the product of the multiplier and autonomous demand (possibly including investment).

In macroeconomics, this insight is applied, among other things, to the problem of unemployment (e.g., Mitchell et al., 2019). This is usually done in the context of the business cycle, i.e., without consideration of the long-run growth potential of the economy as expressed, for example, by productivity growth. However, Keynesian approaches to growth also exist, e.g., Kaldor (1957), Pasinetti (1981), Freitas and Serrano (2015) to name only a few (see Lavoie, 2014 and Blecker and Setterfield, 2020 for an extensive overview).

However, in a dynamic (growth) context, demand may pose a major challenge in terms of stability of the growth path. Imagine, for example, an economy where wage-earners consumer a larger share of their current income than profit earners, and in which wages adjust in the labour market. In such a context, a shock that drives unemployment up will tend to decrease wages, and lower overall demand (through the different propensities to consume). This will tend to amplify the initial shock, driving the economy further away from the normal utilization rates (of labour and capital), in a downward spiral towards sustained (mass) unemployment.

In the empirical reality, we observe such instability in limited periods of depression, not in epochal eras. For example, in the USA over the period 1948 to the first quarter of 2021, the monthly unemployment rate never fell below 3.8% or rose above 14.8%, with the average at 5.8%. Similarly, since 1967, the capital utilization rate moved between the extremes of 64.2% and 89.4%, at an average of 80%.[1] These bounds imply sizeable short- to medium-run fluctuations, but not secular instability. Hence a dynamic theory of demand and growth must reflect this relative stability. Our aim in this paper is to develop a growth model in which demand plays a major role (i.e., we model a multiplier-based economy) and in which both the employment rate and the capital utilization rate show long-run stability. The main version of our model has no government, i.e., a stable growth path is obtained through the dynamics of private behaviour alone. In an extension, we consider the role of the government in stabilizing the growth path.

We choose the Sraffian supermultiplier (SSM, Freitas and Serrano, 2015) as the basis for the model. In this model, investment is endogenized as a fraction of GDP, which implies

[1] Data taken from the FRED database at https://fred.stlouisfed.org/. The indicated starting dates are the earliest for which data is available.

that the multiplier becomes larger, *ceteris paribus*, as compared to the simple Keynesian model where investment is exogenous in the short run. In the simple Keynesian model, the multiplier is equal to $1/(1-c)$, where c is the marginal propensity to consume, while in the supermultiplier model, the multiplier becomes $1/(1-c-h) > 1/(1-c)$, with h as the (time-varying) propensity to invest.

The choice for the supermultiplier model is primarily a convenience choice: the SSM model already generates a stable growth path for the rate of capacity utilization (through endogenous adjustments of h), and thus only the task of modelling a stable path for the employment rate remains. This is, however, not a trivial task, because, as we will show below in detail, the basic SSM model can only generate a stable employment rate if the exogenous rates of growth of labour productivity and of autonomous spending are equal to each other (provided that labour supply does not adjust). This is reminiscent of the notion of Harrodian instability, because these two exogenous rates will only equalize by chance.

Productivity growth (especially labour productivity growth) is an important source of economic growth (Maddison, 1991) that is usually considered as a supply-side force. In turn, technological change is seen as an endogenous driver of productivity. This puts Research and Development (R&D) at the center of analysis (e.g., Aghion and Howitt, 1992 in the mainstream tradition, or Nelson & Winter, 1982 and Silverberg and Verspagen, 1994 in the evolutionary tradition).

Many R&D-based growth models (certainly all of the ones cited in the paragraph above, although we will consider a few exceptions in the next section) ignore the demand side of the economy. Usually, they simply assume that output is equal to capacity output, which, implicitly, calls Say's Law to working. Our model has R&D-based productivity growth, but we also explicitly model the way in which demand adjusts in the long run. This means that demand plays an important role in growth (without demand adjustment no stable growth rate exists), but that the magnitude of the growth rate is determined by supply-side parameters related to R&D and technical change.

The key question that the model poses is whether demand adjustment and productivity growth will simultaneously yield a stable long-run employment path. If demand grows persistently slower than productivity, the economy will tend towards a zero employment rate, whereas if demand grows persistently faster than productivity, labour will become a constraint for growth. In our model, demand and productivity growth are seen to adjust to each other, and a dynamic macroeconomic steady state emerges in which the (super)multiplier is the main economic coordination mechanism rather than price flexibility (Meijers et al., 2019).

In terms of the flow of our exposition, we first extend the basic SSM model by including employment and productivity. We then semi-endogenize[2] productivity with R&D as the main driving factor, and fully endogenize autonomous demand. The central question that the model tries to address is how, and under which parameter settings, demand, both autonomous (i.e., not dependent on current income) and non-autonomous, and

[2] By semi-endogenization we mean that productivity growth depends in an indirect way on a parameter which influences the relative accumulation of fixed capital and R&D capital.

productivity growth will adjust to each other to produce a meaningful and stable steady state growth path.

After this introduction, we present a brief literature review in section 2. Section 3 presents our model. Subsections of Section 3 present the baseline model from Freitas & Serrano (2015), our proposal to endogenize (the growth of) autonomous demand and semi-endogenize technological change (productivity growth), the steady state solutions to our model, stability analysis, and several extensions of our main model, including a rudimentary way to deal with returns to financial investments, and a government stabilization mechanism. In section 4 we summarize the main arguments. More technical details on the model can be found in the appendices.

2. A brief review of some relevant literature

Adjustment of the employment rate plays a central role in our research question, and in the model that is developed in the next section. Here we briefly review a number of recent approaches to labour market adjustment in demand-led growth models. We limit ourselves to models in the supermultiplier tradition, because this is the specific context in which our own model is developed.

The employment rate is the central feature of the labour market that we are interested in. Changes in the employment rate, and whether or not it will be stable in the longer run, depends on the rate of growth of output, the rate of productivity growth, and the rate of growth of labour supply. A stable employment rate requires that the growth rate of labour productivity plus the growth rate of labour supply matches the rate of growth of output. Each of these factors has received attention in the demand-led growth (supermultiplier) literature.

In the supermultiplier tradition, the growth rate of output is equal to the growth rate of autonomous consumption spending (this will be illustrated formally in the next section). In Serrano (1995b, fn. 9, p. 15-16) autonomous consumption can take a wide variety of forms, it includes "the consumption of capitalists; the discretionary consumption of richer workers that have some accumulated wealth and access to credit; residential 'investment' by households ; firms' discretionary expenditures … that do not include the purchase of produced means of production such as consultancy services, research & development, publicity, executive jets, etc. …; government expenditures (both consumption and investment); and total exports."

Most of the items on this long list have been followed up in more recent literature. For Freitas and Serrano (2015), autonomous consumption is just "financed by credit" (p. 261). Pariboni (2016) develops the idea of credit-financed (autonomous) consumption in full. Lavoie (2016) develops the idea of capitalists' consumption, while Nah & Lavoie (2017) focus on exports, and Allain (2015) covers government expenditures. Allain (2019) adds subsistence consumption including an unemployment benefits system to the list. Caminati & Sordi (2019) and Deleidi & Mazzucato (2019) have proposed government R&D as the autonomous expenditure mechanism. However, all these model expansions consider autonomous expenditure as an exogenous variable, as in the basic supermultiplier model that we will outline in the next section.

Of more interest to our approach are attempts to endogenize autonomous consumption (or "non-productive" expenditures by firms). Brochier & Silva (2019) link autonomous consumption to the accumulated wealth of the workers, which is an approach that we will follow. Their model is simulated instead of solved analytically, and there are additional mechanisms influencing the dynamics of the employment rate, which means it is difficult to isolate the impact of endogenizing autonomous consumption by workers' wealth.

Turning to the rate of productivity growth, Fazzari et al. (2020), Nah & Lavoie (2019b), and Palley (2019) all have endogenous labour productivity adjustment, which leads to convergence of productivity growth to the exogenous growth rate (of autonomous spending). Palley (2019) assumes that productivity is sensitive to the employment rate. When the employment rate increases, productivity growth increases, because of learning effects, and when the employment rate falls, the productivity growth rate falls accordingly. With a falling employment rate, productivity growth may become negative in this approach. Fazzari et al. (2020) who also adopt this particular assumption, discuss a host of justifications for this specification.

Caminati & Sordi (2019) introduce R&D as a source of productivity growth. In their approach, the potential of a given amount of R&D to generate productivity growth decreases with the already-achieved level of productivity, and there are also decreasing returns to productivity-adjusted R&D. Because they assume that the amount of (productivity-adjusted) R&D is exogenous, labour productivity grows at a fixed rate. Deleidi & Mazzucato (2019) also have R&D in their model. In this case, business R&D expenditures depend on government R&D.

These R&D-based models, including the model we will present below, are rooted in the neo-Schumpeterian evolutionary tradition, which mainly looks at technology as a supply phenomenon (something that it shares with the mainstream growth theory). However, this tradition criticizes the static orthodox framework of the Walrasian general equilibrium and proposes a new theory for economic microeconomic dynamics with bounded rationality and innovation as a core factor (Nelson and Winter, 1982; Silverberg and Verspagen, 1994). The basis of growth resides in the market implementation of new technologies, in a scenario of competition through innovation, but in a disequilibrium setting. The system follows a process of natural selection, in which the best adapted firms remain in the market.

Hanusch & Pyka (2007) call for a focus of the neo-Schumpeterian literature on the uncertain developments in the socio-economic system, observing the effects of productive transformation also on other aspects (such as the public and monetary aspects) being "*concerned with the conditions for and consequences of a removal and overcoming of the economic constraints limiting the scope of economic development*." (Hanusch & Pyka, 2007, p.276). In our view, this includes the modelling of the demand side of the economy in the form of a multiplier-based model, i.e., the rejection of Say's law (Meijers et al, 2019). This is a prime motivation for our model that combines (semi-) endogenous technological change (productivity growth) and a (super)multiplier mechanism as the main form of coordination in the economic system.

The inclusion of productivity growth is also often done by adding a Kaldor-Verdoorn learning effect (Allain, 2019; Brochier, 2020; Deleidi & Mazzucato, 2019; Nah and Lavoie, 2019b), in which, because of learning by doing, productivity growth depends (positively) on the capital accumulation rate or on the growth rate of output. However, the inclusion of the Kaldor-Verdoorn effect, even if it includes an exogenous component of productivity growth, has only a transitory effect on the capital accumulation rate (which is equal to the growth rate of the economy), unless, as in Nah and Lavoie (2019b), the investment equation is modified to include productivity growth. Kaldor-Verdoorn is neither a necessary nor a sufficient condition to guarantee employment stability. For example, Nah and Lavoie (2019b, p. 289) conclude that "it can (…) be seen (…) that an increase in the autonomous component of technical progress (…) will lead to a one-to-one fall in the long-run growth rate of employment. (…) [T]his conclusion can only be evaded (…) if we assume that faster technical change also generates an increase in the growth rate of the non-capacity-creating autonomous components of effective demand."

Finally, the supply of labour has also been proposed as endogenous. Fazzari et al. (2020) and Nah & Lavoie (2019a) argue that labour supply growth reacts to the employment rate, with high (low) employment rates causing faster (slower) growth of the labour force. Such a mechanism may arise if people base their decision to enter the labour market on the perceived probability of finding a job. With labour supply endogenized in this way, the employment rate will converge to a stable value when the economy grows at the exogenous rate of autonomous spending.

3. The model

We will now present our model in a step-by-step fashion. The first step (sub-section) will be to show how the basic SSM model (Freitas and Serrano, 2015) cannot produce a stable path for the employment rate. After this, we will introduce some new mechanisms to the basic SSM model: first semi-endogenous productivity growth, and then (fully) endogenous growth of autonomous spending. With those two additions in place, we can derive the steady state and investigate its stability. Finally, we present two extensions: one that introduces a rate of return to financial investment, and another that introduces government as a stabilizing force. We conclude the model analysis by investigating stability of the growth path with these extensions in place.

The models that we present are all for a closed economy, and, except for section 3.7, we assume that there is no government.

3.1. Exogenous growth

The key characteristic of the supermultiplier approach is the role of autonomous spending, which we will model as autonomous consumption (but see our review of alternative interpretations in the previous section). Autonomous consumption appears in the consumption function. Contrary to the basic SSM model, we will distinguish between wage-income and profit-income, because this is a foundation of our later extensions of the model. The consumption function thus makes a distinction between the two sources of income:[3]

$$C = \sigma Y c_w + (1 - \sigma) Y c_p + Z_w + Z_p \qquad (1)$$

where C is consumption, the parameter c is a marginal consumption rate, Y is GDP, σ is the share of wage income in GDP (we will consider this as a parameter, although we would think that further development of the model would endogenize this), Z is autonomous consumption (i.e., independent of current GDP) and is exogenous for now, and the subscripts w and p represent wage income and profit income.

An important part of the model is how GDP is determined in the short run. Besides the consumption function, this also requires the investment function, which is simply

$$I = hY$$

where I is investment (the only capacity-creating variable), and h is the propensity to invest (and is an induced variable), which is an endogenous variable in the model for which we will specify an equation below. GDP is then determined by a Keynesian multiplier process, which becomes clear if we derive GDP in the well-known way:

$$Y = C + hY = \sigma Y c_w + (1 - \sigma) Y c_p + Z_w + Z_p + hY \Rightarrow$$

$$Y = \left(Z_w + Z_p\right) \frac{1}{1 - \sigma c_w - (1 - \sigma) c_p - h} \qquad (2)$$

where $1/\left(1 - \sigma c_w - (1 - \sigma) c_p - h\right)$ is the supermultiplier. Note that, contrary to the usual or "normal" multiplier, the supermultiplier includes the share of investment of GDP (the variable h).

To make our model resemble that of Freitas and Serrano, we can assume $c_w = c_p \equiv c$ and define $1 - c \equiv s$. We then also drop the distinction between Z_w and Z_p, and denote total autonomous consumption by Z. Then the supermultiplier becomes $1/(s - h)$ and equation (2) reduces to

$$Y = Z \frac{1}{s - h} \qquad (3)$$

The standard approach of the SSM tradition (Freitas and Serrano, 2015) is to model investment as a dynamic function of the capacity utilization rate:

$$\dot{h} = h\gamma(u - \mu) \qquad (4)$$

Here, γ, the speed of adjustment, and μ, the normal long-run capacity utilization rate, are parameters, u is the capacity utilization rate. The latter is defined as follows:

$$u = \frac{Y}{Y_K}, Y_K = \frac{K}{v}$$

[3] Notation (variables and parameters) is tabulated in Appendix 0.

where Y is output (GDP) K is the stock of fixed capital, ν is the normal capital-output ratio, and Y_K is full-capacity output.

Writing the growth rate of Z as $g_Z \equiv \dot{Z}/Z$ (a dot above a variable will denote a time derivative), and still referring to the reduced (Freitas and Serrano) version of the model, we can write the growth rate of GDP in the following way:

$$\frac{\dot{Y}}{Y} \equiv g = g_Z + \frac{\dot{h}}{s-h} \tag{5}$$

This equation allows us to return to the growth context of the model. The original Freitas and Serrano model consists of two equations: (4) and a differential equation for the capital utilization rate u. By its definition, the growth rate of u is equal to the growth rate of GDP (g as in 5) minus the growth rate of the capital stock. We denote the latter as $g_K \equiv \dot{K}/K$. Like Freitas and Serrano, we assume that capital accumulation is a perpetual inventory process, with a fixed depreciation rate δ:

$$\dot{K} = I - \delta K = hY - \delta K \Rightarrow g_K = \frac{hu}{\nu} - \delta \tag{6}$$

With this, we can write the differential equation for the capacity utilization rate:

$$\frac{\dot{u}}{u} = g - g_K = g_Z + \frac{\dot{h}}{s-h} - \frac{hu}{\nu} + \delta \tag{7}$$

As we are interested not only in the capacity utilization rate but also in employment, we will derive a similar expression for the employment rate, which we will denote by E. We will assume a constant labour force and a fixed labour coefficient in the short run, i.e., $a = Y/L$, where a is labour productivity. Then

$$L = \frac{Y}{a} \Rightarrow \frac{\dot{L}}{L} = g - \frac{\dot{a}}{a}$$

We will allow for labour productivity to grow over time, and will set $\dot{a}/a \equiv \rho$. Then the above equation turns into

$$\frac{\dot{E}}{E} = g - \rho = g_Z + \frac{\dot{h}}{s-h} - \rho \tag{8}$$

If we consider g_Z and ρ as exogenous, we already have a full model that consists of equations (4), (7) and (8), with variables h, u, and E. In the steady state of this model, equation (4) dictates that $u = \mu$ so that $\dot{h} = 0$, and it follows (from equations 7 and 8) that $\dot{u}/u = g_Z - g_K$ and $\dot{E}/E = g_Z - \rho$. This implies that for any steady state values of u and E to exist, we must have

$$g_Z = g_K = \rho \tag{9}$$

In this equation, which is in line with the analysis in Palley (2019) and Fazzari et al. (2020), g_K is endogenous, but (so far) g_Z and ρ remain exogenous. If, like Freitas and Serrano, we set $\rho = 0$, and disregard equation (8) (i.e., do not consider employment to be a variable of interest)[4], then equation (7) solves for $h = (g_Z + \delta)\nu/\mu$. In this steady state, g_K will adjust to become equal to g_Z, and hence the first equality in (9) is satisfied.

But what about the second equality? A (positive) steady state value for the rate of employment, E, requires equality of the two exogenous growth rates ($\rho = g_Z$). If this is satisfied, g_K will adjust to become equal to $\rho = g_Z$. However, if $\rho \gtrless g_Z$, and with both of these rates exogenous, there is no chance of a steady state with both capital utilization and employment constant (and positive).

Because there is no a priori good reason why in a model of exogenous growth $\rho = g_Z$, we proceed to endogenize both these rates. The task is not only to specify how these two rates are equalized, but also whether (and how) the steady state that results from equalization will be stable. Here we do not take the notion of a stable steady state as an approximation of economic reality, as we know well enough that growth paths (including employment rates) are seldom smooth steady states in actual economic history. Instead, we look at the stable steady state that our model looks after as a baseline economic mechanism upon which we must ultimately seek to add turbulence by means of additional economic factors that will remain unspecified in our current analysis.

3.2. Semi-endogenizing the rate of labour productivity growth

In the model with semi-endogenous productivity growth, we start by assuming that a share τ of GDP is spent (out of profit income) on Research and Development (R&D), which we denote by Θ:

$$\Theta = \tau Y$$

We consider τ as an exogenous parameter, but propose that further development of the model would endogenize this parameter. In this way, R&D is modelled similar to investment I, although we consider R&D as non-capacity-creating expenditures (in the strict sense[5]) that are also non-autonomous. We assume that R&D (and the resulting innovation) does not affect the quality of capital, as it would in a vintage model (e.g., Silverberg and verspagen, 1994).

With R&D included in the model, equation (2) changes to

$$Y = \left(Z_w + Z_p\right)\frac{1}{1-\sigma c_w-(1-\sigma)c_p-\tau-h} \tag{2a}$$

Productivity change depends on the accumulated knowledge stock (not just on current R&D activities), and hence we introduce an R&D-capital stock (denoted by R). We follow the empirical literature that addresses the relation between R&D and productivity (see, e.g., Hall et al., 2010) in assuming that R evolves as a stock in the same way as fixed capital

[4] Not considering employment not only assumes that labour is not a constraint to growth, but also, and perhaps more importantly, that unemployment is not a factor of interest. Palley (2019), Brochier (2020) and Fazzari et al. (2020) are exceptions in the SSM literature who recognize the importance of including employment in the model.

[5] We follow the existing literature (see Section 2) in calling R&D non-capacity-creating, although it clearly enhances the capacity of labour to produce output.

K, with a fixed depreciation rate Δ (for most of our steady state calculations, we will assume $\Delta = \delta$, i.e., R&D capital and fixed capital depreciate at the same rate, which will simplify the mathematics):

$$\dot{R} = \Theta - \Delta R = \tau Y - \Delta R \tag{10}$$

Further, we define the ratio of the R&D-capital stock to the stock of fixed capital as

$$\Phi \equiv R/K$$

Using (6) and (10), we get

$$\dot{\Phi} = \Phi\left(\frac{\tau u}{\Phi v} - \Delta - \frac{hu}{v} + \delta\right) = \Phi\frac{u}{v}\left(\frac{\tau}{\Phi} - \Delta - h + \delta\right) \tag{11}$$

Finally, we again follow the empirical literature in assuming that productivity growth results from R&D intensity (in our case intensity relative to the capital stock), more specifically, from the value of Φ:

$$\rho = \bar{\rho} + \varphi\Phi \tag{12}$$

where $\bar{\rho}$ (the exogenous part of productivity growth) and φ are parameters. Note that if we set $\tau = 0$, it follows that $\Phi = 0$, or if we set $\varphi = 0$, we are back in the realm of completely exogenous productivity growth ($\rho = \bar{\rho}$).

3.3. Endogenizing the growth rate of autonomous spending: the private sector[6]

We will now set out to fully endogenize autonomous spending. This means that demand will become fully endogenous to the model (non-autonomous demand is already fully endogenous). With productivity growth semi-endogenous, i.e., ultimately dependent on the rate of R&D investment τ, this implies that the magnitude of the growth rate will become dependent on the R&D parameters (which are all supply side). However, demand will still play an important role in the model because without demand adjustment, the growth path cannot be stable.

The first idea that we will employ for endogenizing g_Z is that private autonomous consumption depends on accumulated wealth. This is the foundation of the first (and main) model that we present, while in a later subsection we will consider government fiscal policy as another source of endogenous autonomous spending in the economy in an alternative model.

In the appendix, we present a model where, in line with the general consumption function (2), we distinguish between consumption out of wage income as well as profit income, and accumulated wealth is also from wage income and accumulated wealth from profit income. For clarity in exposition, here, in the main text, we will focus on a special case, where there is no consumption out of profit income, nor out of accumulated wealth out of profits. In other words, only wage income and the accumulated savings out of wage

[6] We understand an endogenous variable as a variable that depends on other variable(s) in the model, while an autonomous variable is one that is independent of current GDP. In our endogenization of g_Z, it remains independent of current GDP, i.e., it remains autonomous, which is in line with the SSM tradition.

income are used for consumption. In this special case, the basic outcomes of the model in terms of growth are unchanged relative to the more general appendix model, and the formal expressions for steady state values are significantly simplified relative to the general case.

The basic idea for endogenizing g_Z is taken from Brochier (2020), and states that autonomous consumption is dependent on accumulated wealth. In the specific case that we consider here in the main text, this wealth is defined as accumulated savings purely out of labour income (wages). The appendix deals with the more general model in which also accumulated wealth out of profit income is considered.

We denote accumulated savings out of labour income by W_w, and specify its motion by the following equation:[7]

$$\dot{W}_w = (1 - c_w)\sigma Y - Z_W - \delta W_w \quad \text{(A2a)}$$

The first part of the righthand side $((1 - c_w)\sigma Y - Z_W)$ simply represents savings out of current labour income. The term $-\delta W_w$ represents depreciation of wealth. This arises from the specific setup of the model, explained in more detail in the appendix, in which total accumulated wealth in the economy is equal to the sum of the productive capital stocks (both R&D capital R and fixed capital K). In this way, W_w is seen as an entitlement of the holders (wage earners) on the stock $R + K$. The term $-\delta W_w$ is included because the entitlement to $R + K$ will depreciate with the stocks themselves, and we assume, for simplicity, that R&D capital and fixed capital depreciate at the same rate $\Delta = \delta$.

The general model has a corresponding wealth variable W_p, which represents assets held by profit earners. As the appendix shows, $W_w + W_p = R + K$. Because of the specific assumptions made here (no autonomous consumption out of W_p), we do not need the variable W_p in the exposition in the main text. However, we do have to introduce the variable $x \equiv W_w/\left(W_w + W_p\right) = W_w/(K + R)$, which represents the share of wage earners in total wealth of the economy.

The endogenization of g_Z then proceeds by positing

$$Z_w = \zeta_w W_w \quad \text{(A4a)}$$

$$Z_p = \zeta_p W_p \quad \text{(A4b)}$$

Here ζ_w is a new variable that represents the (marginal) propensity to consume out of accumulated workers' savings, and similarly ζ_p is a parameter that represents the marginal propensity to consume out of profit earners' assets. Note that we assume $\zeta_p = 0$ (as well as $c_p = 0$) in the main text, i.e., equation (A4b) is only reported here for completeness (these assumptions are relaxed in Appendix 1). Our assumption is also that the variable ζ_w is a behavioural variable that serves to smooth (autonomous)

[7] Equation numbers starting with A refer to one of the three appendices. These equations are introduced and discussed in some detail in the appendices, in this case the context of the model with generalized consumption function.

consumption spending for changes in workers' income that result from changes in the employment rate. More specifically, we specify

$$\dot{\zeta_w} = \iota\zeta_w(\bar{E} - E) \tag{13}$$

Here ι and $\bar{E}$ are parameters (both >0). $\bar{E}$ specifies a neutral rate of employment at which current wage income is considered satisfactory. When the employment rate drops below $\bar{E}$, current labour income also falls below the satisfactory level (remember we assume a fixed real wage rate), and workers have to "compensate" by drawing to a larger extent on their accumulated wealth for consumption. This means that ζ_w will have to rise. Similarly, when employment rises above $\bar{E}$, labour income is considered high, and there is less of a need for consumption out of accumulated wealth. Hence ζ_w will fall. We adopt the shorthand term "consumption smoothing" (James et al., 2007; Kim et al., 2014) for the idea specified by equation (13), which is a key mechanism in our model that proves to provide stability to the growth path in terms of ensuring a stable employment rate.

It is easy to see how equation (13) has the potential to stabilize the economy. If employment falls below the neutral value ($\bar{E}$), autonomous consumption will tend to increase (W_w will be fixed initially, while ζ_w increases), and ceteris paribus the multiplier, GDP will increase, bringing the employment back towards the neutral rate $\bar{E}$. Note that such stabilization works exclusively through quantity adjustment (of autonomous demand). Because demand in our model depends largely on consumption out of wage income, it is hard to imagine how flexible prices would achieve stabilization. For example, a traditional Phillips curve regulating (real) wages would have a de-stabilizing effect, because wages (and hence demand) would fall when employment falls below a threshold level, leading to a spiral that bring the economy further away from a stable employment rate.

The ultimate model (with semi-endogenized ρ and fully endogenized g_Z) contains six variables: h, u, E, Φ, ζ_w and x. We have already specified differential equations for h, Φ and ζ_w (equations 4, 11 and 13, respectively). We also have general forms (equations 7 and 8) for the differential equations for u and E, in which we still need to specify the endogenized variable g_Z. We also need to specify the differential equation for x.

Let us start by writing the expression for g_Z, which will give us two differential equations. Clearly, from equation (A4a), $g_Z = \left(\dot{\zeta_w}/\zeta_w\right) + \left(\dot{W_w}/W_w\right)$. The first of the terms on the righthand side of this follows directly from equation (13). With the assumption that autonomous consumption out of profit income is zero, we can also write

$$g_W \equiv \frac{\dot{W_w}}{W_w} = \zeta_w x \left(\frac{\tau + h}{1 - c_w\sigma - \tau - h}\right) - \delta \tag{14}$$

(This equation is a specific case of equation A6). This leads to

$$g_Z = \iota(\bar{E} - E) + \zeta_w x \left(\frac{\tau + h}{1 - c_w\sigma - \tau - h}\right) - \delta \tag{15}$$

(the more general form of this is A7).

And then:

$$\dot{u} = u\left(\iota(\bar{E} - E) + \zeta_w x\left(\frac{\tau+h}{1-c_w\sigma-\tau-h}\right) + \frac{\dot{h}}{1-c_w\sigma-\tau-h} - \frac{hu}{v}\right) \tag{7'}$$

$$\dot{E} = E\left(\iota(\bar{E} - E) + \zeta_w x\left(\frac{\tau+h}{1-c_w\sigma-\tau-h}\right) + \frac{\dot{h}}{1-c_w\sigma-\tau-h} - \delta - \bar{\rho} - \varphi\Phi\right) \tag{8'}$$

Finally, we can derive the last differential equation from the definition $x = W_w/(W_w + W_p)$:

$$\dot{x} = \frac{(\zeta_w x+\zeta_p(1-x))((1-c_w)\sigma-x(\tau+h))}{1-c_w\sigma-c_P(1-\sigma)-\tau-h} - \zeta_w x \tag{16}$$

3.4. Steady state

In this section, we will analyze the steady state of the model with semi-endogenous productivity growth and endogenous g_Z as specified in the previous section (i.e., private autonomous spending as the source of g_Z). We will ignore the trivial steady state solution where all variables are zero.

We can start by setting equations (4) and (13) to zero, which immediately yields $u^* = \mu$ and $E^* = \bar{E}$ (we will denote steady state solutions by a * superscript). Next, setting equations (7') and (8') to zero, re-arranging and equating the results of both equations, as well as substituting (15), yields the following quadratic equation for Φ:

$$\varphi\Phi^2 + (\delta + \bar{\rho})\Phi - \frac{\tau\mu}{v} = 0 \tag{17}$$

Obviously, this equation has two solutions, but only one of them yields a positive value for Φ:

$$\Phi^* = \frac{-(\delta+\bar{\rho})+\sqrt{(\delta+\bar{\rho})^2+\frac{4\varphi\tau\mu}{v}}}{2\varphi} \tag{18}$$

To derive the steady state value for h, two routes are now open. One of these is to set equation (11) to zero, which yields (remember we assume $\Delta = \delta$ for simplicity):

$$\dot{\Phi} = \Phi\frac{u}{v}\left(\frac{\tau}{\Phi} - h\right) = 0 \Rightarrow \frac{\tau}{\Phi^*} = h^* \tag{19}$$

Equation (18) could be substituted into (19) to obtain h^* as a function of only parameters. The other route is to set (7') and (8') to zero, solve each of these for ζ_w and equate the two expressions, which yields

$$h^* = \frac{v}{\mu}(\delta + \bar{\rho} + \varphi\Phi^*) \tag{20}$$

Again, we can substitute (19) to obtain an (alternative) expression for h^* as a function of only parameters. Equation (20) also shows that the steady state value for the investment rate h depends on the rate of technological change (or productivity growth): as productivity growth rate is faster, capital needs to accumulate at the same rate (see equation 9), which requires a higher value of h.

However, rather than actually presenting either one of these expressions (which are equivalent), we show in Figure 1 how the parameter τ influences the simultaneous determination of the steady state solutions Φ^* and h^*, as well as the resulting rate of productivity growth, ρ. The figure shows that all three of these steady state outcomes are concave functions of τ. In other words, increasing the rate of R&D spending as a fraction of GDP will have a positive but declining effect on the steady state values of the investment rate (h), the ratio of the R&D stock to the stock of fixed capital (Φ), and, as a result of the latter, the rate of productivity growth.

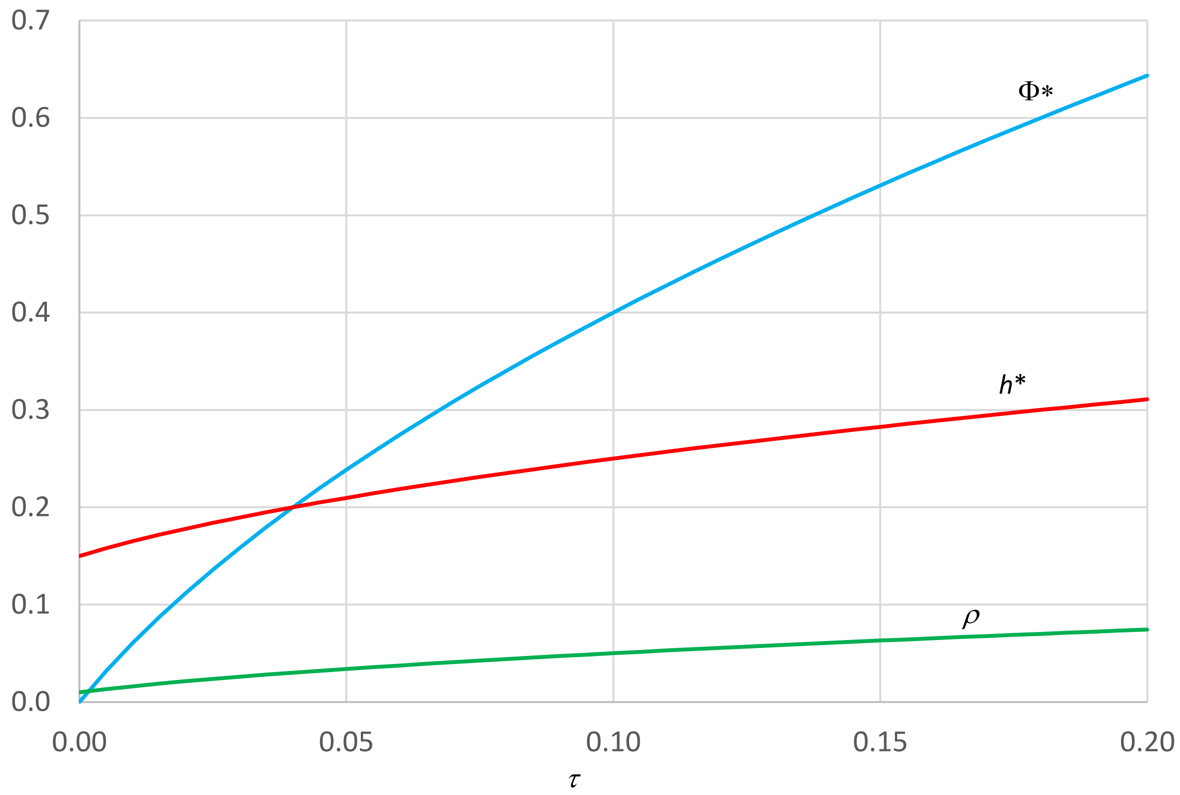


Figure 1. Steady state values of Φ, h and ρ as a function of τ

Next, we set equation (16) to zero, solve for ζ_w and equate this to the expression for ζ_w that can be derived from setting (8') to zero. This yields the steady state solution for x:

$$x^* = 1 - \frac{(1-\sigma)}{(\tau+h^*)} \tag{21}$$

Obviously, the denominator of the fraction on the righthand side $(\tau + h^*)$ is the fraction of GDP that needs to be invested (in fixed capital and R&D) in the steady state. The higher this share is, the more firms need to borrow from workers to fund investment, and hence the higher the steady state value of x.

Finally, we can use any of the expressions that were derived for ζ_w and substitute x^* to obtain

$$\zeta_w^* = \frac{\left(1-\sigma c_w-(\tau+h^*)\right)h^{*\frac{\mu}{v}}}{(\tau+h^*)-(1-\sigma)} \tag{22}$$

In this expression, as long as $\tau + h^* < 1$, σ has a negative effect on ζ_w^* (the higher the share of wages in GDP, the lower the resulting rate of autonomous consumption spending). The effect of τ is also negative, which is in line with our conclusion on equation (21) (the higher τ, the more firms tend to borrow). The effect of h^* is harder isolate, but it appears to be negative, with the same intuition as the effect for τ.

With all steady state values of the endogenous variables derived, we are able to look at how growth works in this model. Equation (9) states that $g_Z = g_K = \rho$, and we also know that in the steady state the growth rate of GDP (g) is equal to this. Using equation (12), we see that in the steady state $\rho = \bar{\rho} + \varphi\Phi^*$, and equation (18) shows that Φ^* depends on a range of parameters, which includes δ, τ, μ, and v ($\bar{\rho}$ and φ were already included).

As was already predicted above, all of these are supply-side parameters, and some ($\tau, \bar{\rho}$ and φ) are directly related to technological change. Demand-side parameters, such as c_w, $\bar{E}$ or even σ do not enter the expression for the long-run growth rate of the economy. What happens is that g_Z (the demand side) adjusts to the growth rate of productivity. This also implies that the endogenization of the demand side (g_Z) is crucial for the existence and stability of a steady state. In fact, we could keep the rate of productivity growth completely exogenous (e.g., $\varphi = 0$), and, as long as we keep the endogenization of g_Z, the steady state of the model would still exist.

In the next section, we will consider whether demand side adjustment can produce a stable path towards the steady state values that we derived. Before we undertake to answer this question, we can also note that the general model, for which we document the steady state expressions in the appendix, arrives at the same conclusion with regard to the unique importance of the supply side in determining the growth rate. In other words, also if we relax the assumption that only wage earners consume, we see no change in the steady state growth rate of the economy. Only the steady state values of x and ζ_w will change if we relax those assumptions.

3.5. Stability analysis

We used numerical simulations to explore the behaviour of the model as specified so far.[8] These simulations were done in R, using the ssmmod function, which numerically integrates the equations. Figure 2 documents the time paths for the variables of the model in the baseline simulation, which uses the following parameter values: $\tau = 0.05$, $\delta = 0.05$, $v = 2$, $\mu = 0.8$, $\bar{E} = 0.8$, $c_w = 0.875$, $\sigma = 0.8$, $\varphi = 0.1$, $\bar{\rho} = 0.01$, $\iota = 1.4$, $\gamma = 0.15$ as well as $c_p = 0$ and $\zeta_p = 0$ which we assumed throughout the main text so far. We see that, for these parameter values, the model converges (with dampened fluctuations) to the steady state. The steady state values of the variables are as follows: $u^* = 0.8$, $h^* = 0.210$, $E^* = 0.8$, $x^* = 0.230$, $\zeta_w^* = 0.057$, $\Phi^* = 0.239$.

8 For a more detailed calibration of the baseline SSM, see Haluska et al. (2021) for the US economy.

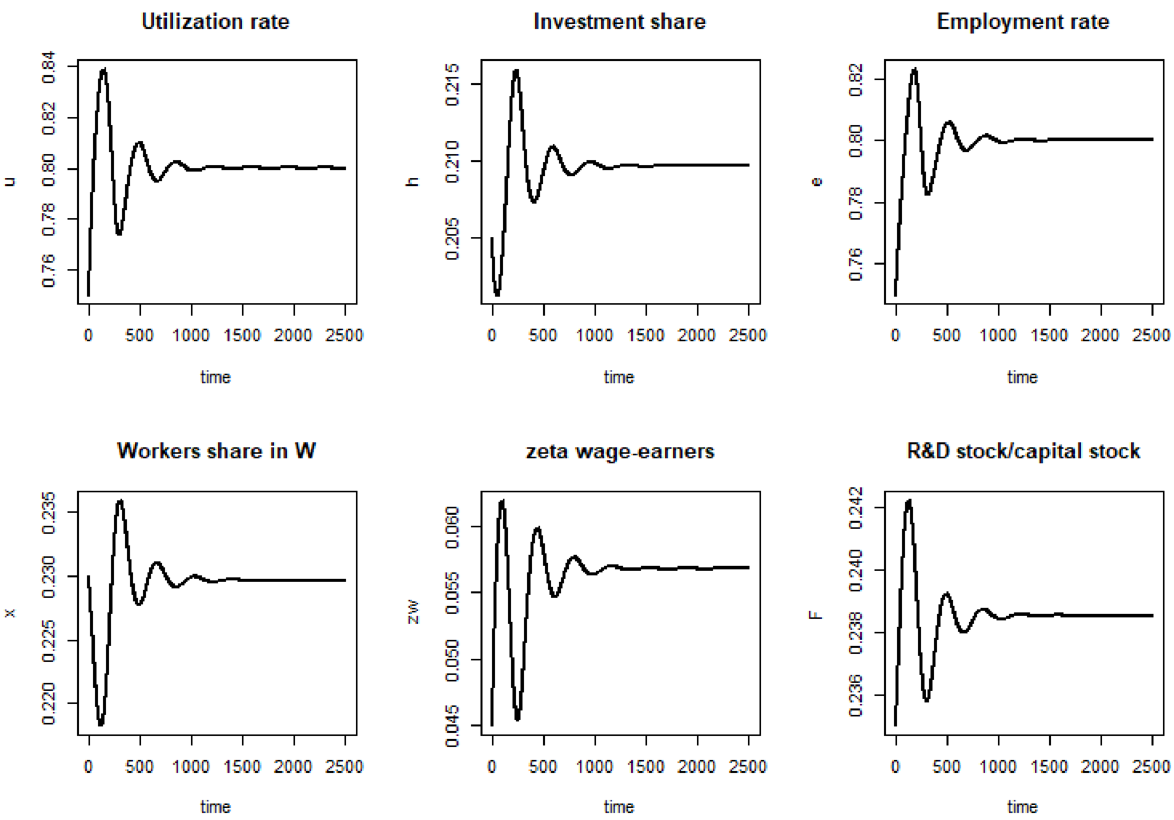


Figure 2. Model simulation showing stability of the steady state

In order to obtain a more comprehensive overview of stability, we used Matlab's symbolic math toolbox to derive the Jacobian matrix corresponding to the model. This enables us to do a grid search of parameter space, and numerically calculate the eigenvalues of the Jacobian matrix at the steady state, for each particular parameter constellation. With 11 (or even 13) parameters, it is impossible to do a complete search. Therefore, we limited our search of parameter space to just 5 parameters, which are τ, σ, c_w, ι and γ. The other parameters are fixed at the values listed above. Note that while we have reasonably good ideas for what are plausible values for some of these parameters (τ, σ, c_w), we do not have such expectations for ι and γ. This is why we choose to put these latter two parameters on the axes in the figures below.

The parameter grid search not only gives information about stability of the steady state. It also provides insights into the sign of some of the steady state values of the variables. In the broad and coarse grid search that we implemented[9], it appeared that there are parameter sets in which either x or ζ_w (or both) are negative (and stable). While such negative values are not impossible to interpret (essentially, they represent an indebted working class), we will focus on parameter values that yield positive values for x and ζ_w. The parameter grid search suggests that we need fairly high values of σ (typically 0.75 or higher for the restricted model of the main text) and τ (typically 0.05 or larger) to ensure this. Relaxing the assumption that profit earners do not consume reduces the likelihood

[9] We analyzed the following ranges in this broad and coarse grid search τ: 0.01 – 0.07; σ: 0.5 – 0.9; c_w: 0.4 – 0.9, ι: 0.01 – 1.5; and γ: 0.01 – 0.25.

of negative steady state values for x or ζ_w considerably. The key parameter to relax this assumption is ζ_p. If we set $\zeta_p = 0.15$, values of $\sigma = 0.7$ or lower still generate positive values for x and ζ_w.

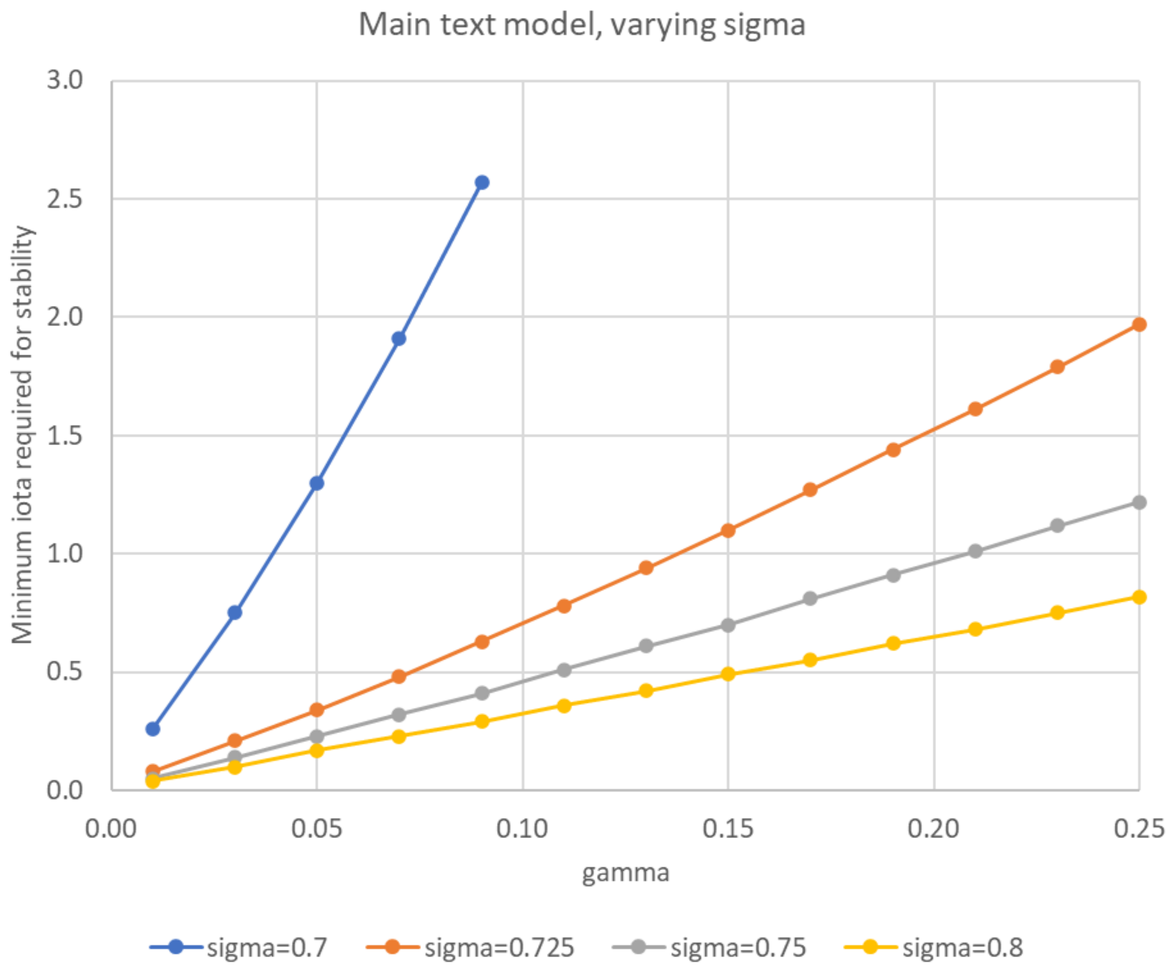


Figure 3. The role of parameters γ and ι in the main model for stability of the steady state

Inspection of the results of the grid search suggests that the parameters ι and γ play a crucial role in stability. In particular, we need a minimum value of the ratio ι/γ for the steady state to be stable. This is illustrated in Figure 3, which documents, for different (and fixed) values of the parameters other than ι and γ, on the vertical axis the minimum value of ι that yields a stable steady state given the value of γ on the horizontal axis.[10] For example, the grey line (which is drawn for a value $\sigma = 0.75$) shows that if $\gamma = 0.15$, stable steady state values are obtained for values of $\iota \geqq 0.7$. All lines in the figure are (approximately) linear, which means that along each line, the ratio ι/γ is fixed, and we

[10] A stable steady state, in this case means that all eigenvalues (of the Jacobian matrix evaluated at the steady state) are either non-imaginary and negative, or imaginary with a negative real part. We also checked for zero real (parts of) eigenvalues, but this did not happen in the cases we considered.

need this ratio to be larger than the slope of the line for the model to have a stable steady state. This implies that ι needs to be relatively large for stability, and that larger values of γ require larger values of ι.

The figure also provides different lines for varying values of σ. These indicate that the lower σ is (e.g., the blue line represents the lowest value of σ that we considered here, 0.7), the higher the required ratio ι/γ is. Given γ, higher values of pose lower restrictions on ι for stability. For example, the individual simulation run that we documented above (with $\sigma = 0.8$, $\gamma = 0.15$ and $\iota = 0.85$) lies well above the yellow line.

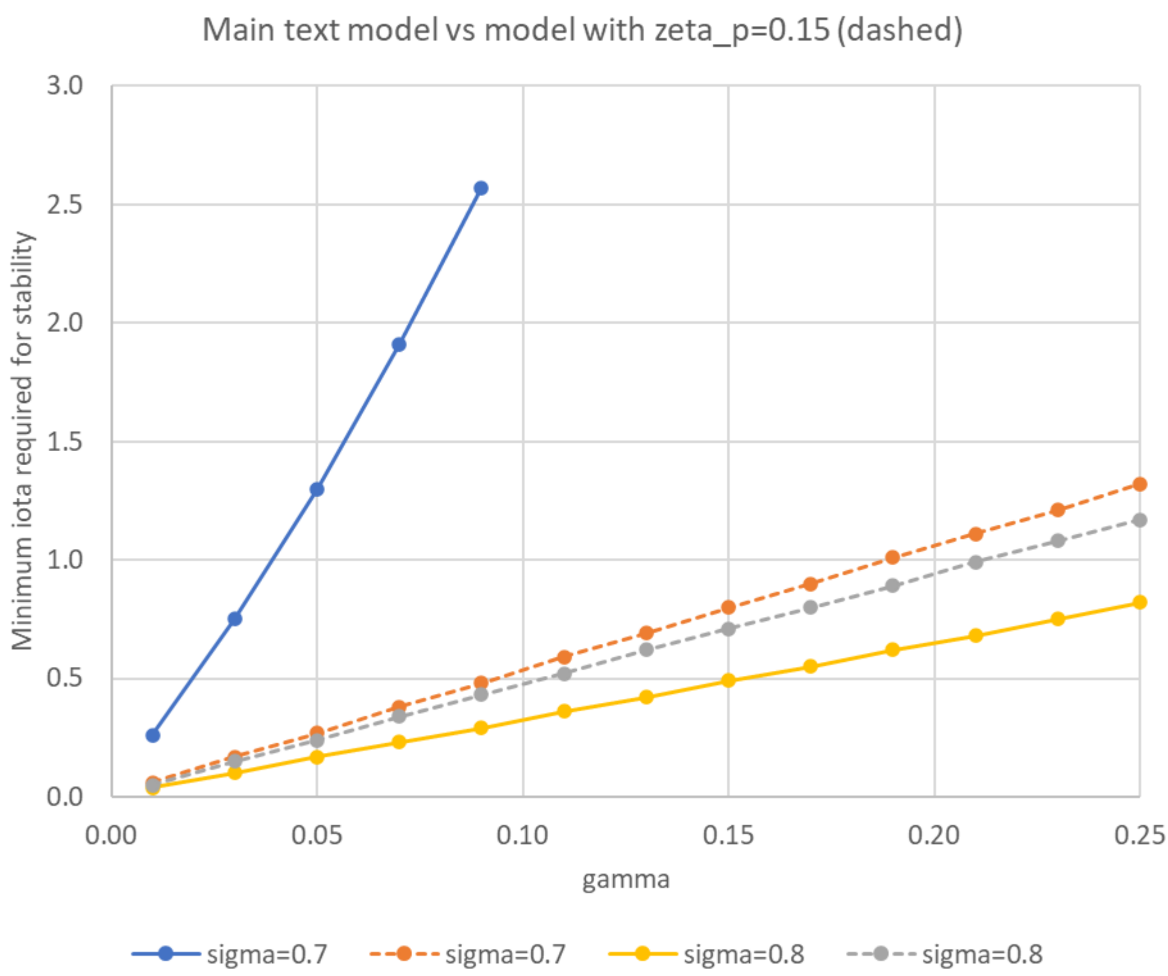


Figure 4. The role of parameters γ and ι in the main model with zero and positive ζ_p for stability of the steady state

Figure 4 documents similar results, but now comparing to a more general case where we relax the assumption $\zeta_p = 0$ and instead set $\zeta_p = 0.15$ ($c_p = 0$ remains as an assumption). Here we only consider the 'extreme' values $\sigma = 0.8$ and $\sigma = 0.9$, i.e., the blue and yellow curves are the same as in the previous figure. Interestingly, compared to the case $\zeta_p = 0$, $\zeta_p = 0.15$ moves the lines ι/γ much closer together. This means that with $\zeta_p = 0.15$, differences in σ matter much less for stability than in the case $\zeta_p = 0$. The intuition for this result lies in the variable x. Because profit earners now consume out of their wealth, their share of total assets in the economy decreases, i.e., the steady state value for x increases. With a larger base for their autonomous consumption, consumption smoothing becomes easier for workers, which makes ι a less crucial parameter for stability.

3.6. Introducing a rate of return to accumulated savings

So far, we have assumed that there is no return on the accumulated savings by workers. This is, of course, an unrealistic assumption. Ideally, the model that we presented so far would be extended to include a financial market, which would offer various instruments that could be used to invest savings. Such a financial sector would also have to include additional agents, such as a government, a central bank and private banks. While the tradition of so-called stock-flow-consistent models (Godley and Lavoie, 2007; Brochier & Silva, 2019) offers such models, we leave the extension of our model in this elaborate way to future work. Instead, we opt here for a very rudimentary way of incorporating a rate of return.

Our main idea, which we already briefly referenced above, is that accumulated workers' savings can be seen as a claim on the production factors that are accumulated in firms, i.e., fixed capital and knowledge capital (R&D). Remember that total assets (aggregated over workers and profit earners) in the economy are equal to the sum of the capital stock and the R&D stock. Workers' assets (accumulated savings) are a share of this (which is the variable x), and our equations will specify that workers are entitled to a proportional share of current profits.

Such an allocation of part of profit income to workers means that workers now receive more (if $x > 0$) than the share σ of GDP. If this redistribution would be completely proportional, workers would receive a share $\sigma + x(1-\sigma)$, and profit earners a share $(1-x)(1-\sigma)$. However, we introduce a new parameter, $0 \leq r \leq 1$, which measures the extent to which profits are redistributed. With the inclusion of this new parameter, workers will receive a share $\sigma + rx(1-\sigma)$ of GDP, and profits earners a share $(1-\sigma)(1-rx)$. It is easily seen that if $r = 0$ (i.e., no returns on accumulated savings), we have the model as it has been presented so far, while if $r = 1$, we have the expressions as firstly introduced in this paragraph (returns fully proportional to x).

The marginal propensities to consume (or save) for workers and profit earners are applied to total income, i.e., to the shares of GDP as specified in the previous paragraph. This changes some of the equations in the model, and this is documented in full detail in Appendix 2. Here, we only summarize the main result of this change, which solely lies in the steady state values for the variables x and ζ_w. The steady state expressions are given in the appendix. Generally, for $r > 0$, we find larger steady state values for x as compared to equation (21), or its more general counterpart found in Appendix 1 (equation A12). For the case $r = 1$, we find $x^* = 1$, i.e., workers own the entire capital stock. For all cases $r < 1$, $x^* < 1$ remains, thus as long as profit-earners are left with some of the current-period profits, they will always be able to accumulate at least some level of positive assets.

However, when $r = 1$, and independently of how much they save, the savings of the profit earners become insufficient to compensate for the depreciation on the physical assets they hold.

The positive effect on x^* in cases $r > 0$ has implications for stability. This is shown in Figure 5, which is similar to the two previous figures. Here all lines are drawn for $\sigma = 0.8$. The solid line is the case of the model of the main text, i.e., $\zeta_p = 0$, which yields the same line as already shown in the previous two figures. The dashed grey line assumes $\zeta_p = 0.15$ but keeps $r = 0$, and hence this is the same line as in the previous figure. The other two lines introduce two new cases: $r = 0.5$ (green) and $r = 1$ (red). We can see that a higher value of r shifts the ι/γ tradeoff line down, i.e., increases stability. This is the same effect as observed before: the increase in x^* makes consumption smoothing easier to implement for given ι.

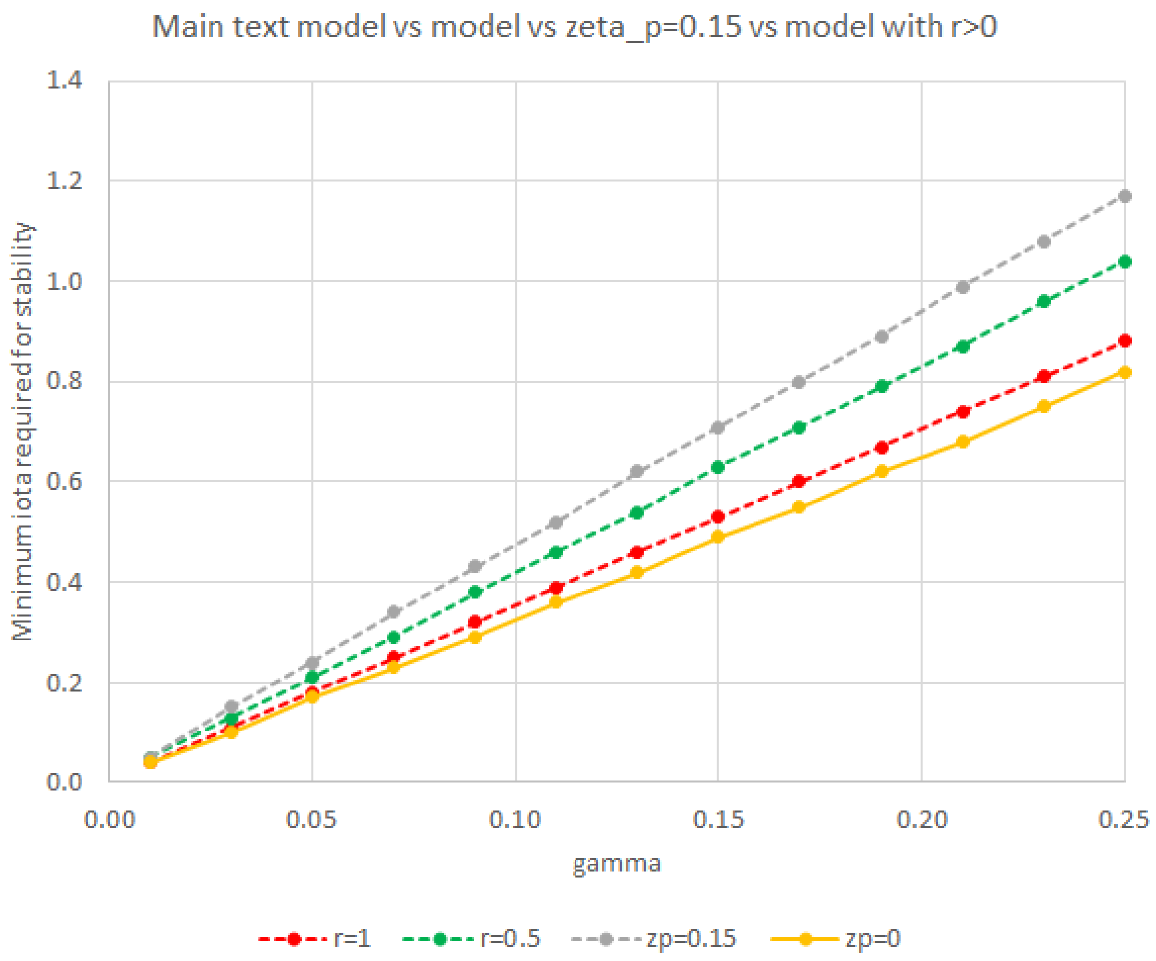


Figure 5. The role of parameters γ and ι in the model with a financial return for stability of the steady state

3.7. Endogenizing the growth rate of autonomous spending: the government sector

We conclude our model analysis by considering an alternative mechanism for stabilizing the economy. Whereas so far private autonomous consumption spending worked as a stability mechanism by workers' desire to smooth consumption between periods of high and low unemployment, we now ask whether the government is able to stabilize the economy by fiscal policy. We limit our model extensions to the government mission of stabilizing the economy, hence we disregard the type of missions that, e.g., Deleidi and Mazzucato (2019) consider. Here, we will discuss the changes that are made to the model of the previous sections to analyze this question. Full details of some of the equations of the model with government fiscal policy are provided in Appendix 3.

In order to consider the model with a government, we will make one major simplification to the model as considered so far: we will no longer distinguish between workers and profit earners in the private sector. This means that the variable x is no longer relevant, and that we have only a single parameter for the marginal consumption rate (we will denote this parameter by c), and a single variable for autonomous spending by the private sector (this variable will be denoted as Z_h). As before, Z_h will be a fraction (denoted by ζ_h) of total private-sector assets (or wealth). Because our focus is on the government sector as a stabilization mechanism, we will assume that ζ_h is a fixed parameter.

On the other hand, the introduction of a government sector also means that we have to introduce new variables and equations into the model. The first of these variables is Z_G, which is autonomous government (consumption) spending. Another variable is the tax rate T, which is specified as a share of GDP, which implies that TY is total tax revenue. We also specify total outstanding government debt, which we denote by G.

In line with the previous section, we assume that the government has to pay interest on the bonds that it issues to fund outstanding debt G. For simplicity, we assume that this rate of return is equal to the private rate of return on invested capital (R&D capital R and fixed capital K). We then consider profit income as the return on invested capital, which means that the rate of return is equal to $(1-\sigma)Y/W$, where σ is, as before, the parameter that represents the share of wages in GDP, and hence $(1-\sigma)$ is the share of profits. W denotes the total invested capital by private agents, which in the model of the previous sections was split into W_w and W_p. Because we have now assumed a fixed ζ_h, we have $W = W_w + W_p = R + K$, where W_w and W_p are variables that are only relevant for the comparison with the model of the previous sections.

Finally, we introduce a new variable $D \equiv G/W$, which is government debt as a share of $W = R + K$. With the interest rate on government bonds equal to $(1-\sigma)Y/W$, total interest payments (to the public, which holds the bonds) are equal to $G\,(1-\sigma)Y/W = (1-\sigma)DY$. Then we have

$$\dot{G} = Z_G + (1-\sigma)DY - TY = Z_G + Y\big((1-\sigma)D - T\big) \quad (23)$$

and

$$\dot{D} = \zeta_G + \frac{[(\zeta_H+\zeta_G)+\zeta_H D][D(1-\sigma-h-\tau)-t]}{1-c(1-t+(1-\sigma)D)-(h+\tau)} + D\delta \quad (24)$$

Here Z_G is autonomous government spending, and ζ_G is the parameter that governs this variable. The government model needs a number of behavioral equations for the government. First, we assume that government spending is proportional (by ζ_G) to the private wealth variable W:

$$Z_G = \zeta_G W \quad \text{(A4d)}$$

We then also need a behavioral rule for the spending fraction ζ_G:

$$\dot{\zeta}_G = \zeta_G \iota_G (\bar{E} - E) \quad \text{(13a)}$$

This is similar to the rule that we used for workers' consumption smoothing in the previous sections (equation 13), but here it is the government who takes this task upon itself. Equation (13a) reflects the government "mission" to stabilize the economy. The main difference with the previous sections is that the government needs to borrow money to perform this function, whereas workers could draw on their savings. Hence the government raises taxes to fund its debt, and therefore we need a behavioral rule for the tax rate. Here, we will assume that the government sets a long-run neutral value for the variable D, and adjusts the tax rate to maintain this value (in the long run):

$$\dot{T} = T\eta(D - \bar{D}) \quad \text{(25)}$$

In what follows, we will set $\bar{D}$ (the neutral D value) to zero, which means that the government aims to have no debt in the long run (the model that we analyze is a balanced budget supermultiplier model). This is a strict assumption, which we make for mathematical convenience, but assuming $\bar{D} > 0$ does not fundamentally change the conclusions.

This concludes the model with government stabilization. The model consists of seven endogenous variables: h, Φ, u, E (all of which were present in the model without a government), T, D, and ζ_G. The differential equations for h and Φ are unchanged, they are (4) and (11), respectively. The differential equations for u and E are slightly changed and are specified by substituting the new expression for g_Z (equation A7a in Appendix 3) into the generic forms (7) and (8). These two equations are documented as (7a) and (8a) in Appendix 3. Finally, equations (25), (24) and (13a) provide the differential equations for the new variables T, D, and ζ_G.

3.8. Steady state and stability

The (non-trivial) steady solution of the model can be derived in the same way as was done for the model without a government. The expressions for h^*, Φ^*, u^* and E^* do not change from what we had without a government, which leaves only the three new government-related variables. We leave details of the derivations of the steady state values of these variables to the interested reader, and just document these values:

$$D = \bar{D} = 0 \quad \text{(26)}$$

$$\zeta_G^* = \frac{\mu}{v(1+\Phi^*)} - \frac{(\delta + \bar{\rho} + \varphi\Phi^*) + \zeta_h}{(1-c)} \quad \text{(27)}$$

$$T^* = 1 - \frac{v(1+\Phi^*)}{\mu}\frac{(\delta+\bar{\rho}+\varphi\Phi^*)+\zeta_h}{(1-c)} \tag{28}$$

We simulate this model with the following parameter values: $\tau = 0.075$, $\delta = 0.05$, $v = 2$, $\mu = 0.8, \bar{E} = 0.8, c = 0.6, \sigma = 0.7, \varphi = 0.1, \bar{\rho} = 0.01, \iota_G = 0.95, \gamma = 0.1, \zeta_h = 0.01$, and $\eta = 0.1$. Figure 6 shows how, under these parameter values, the model converges to the steady state with dampening oscillations. Along the adjustment path, the variable D also takes negative values, i.e., at some times, the government borrows money from the private sector.

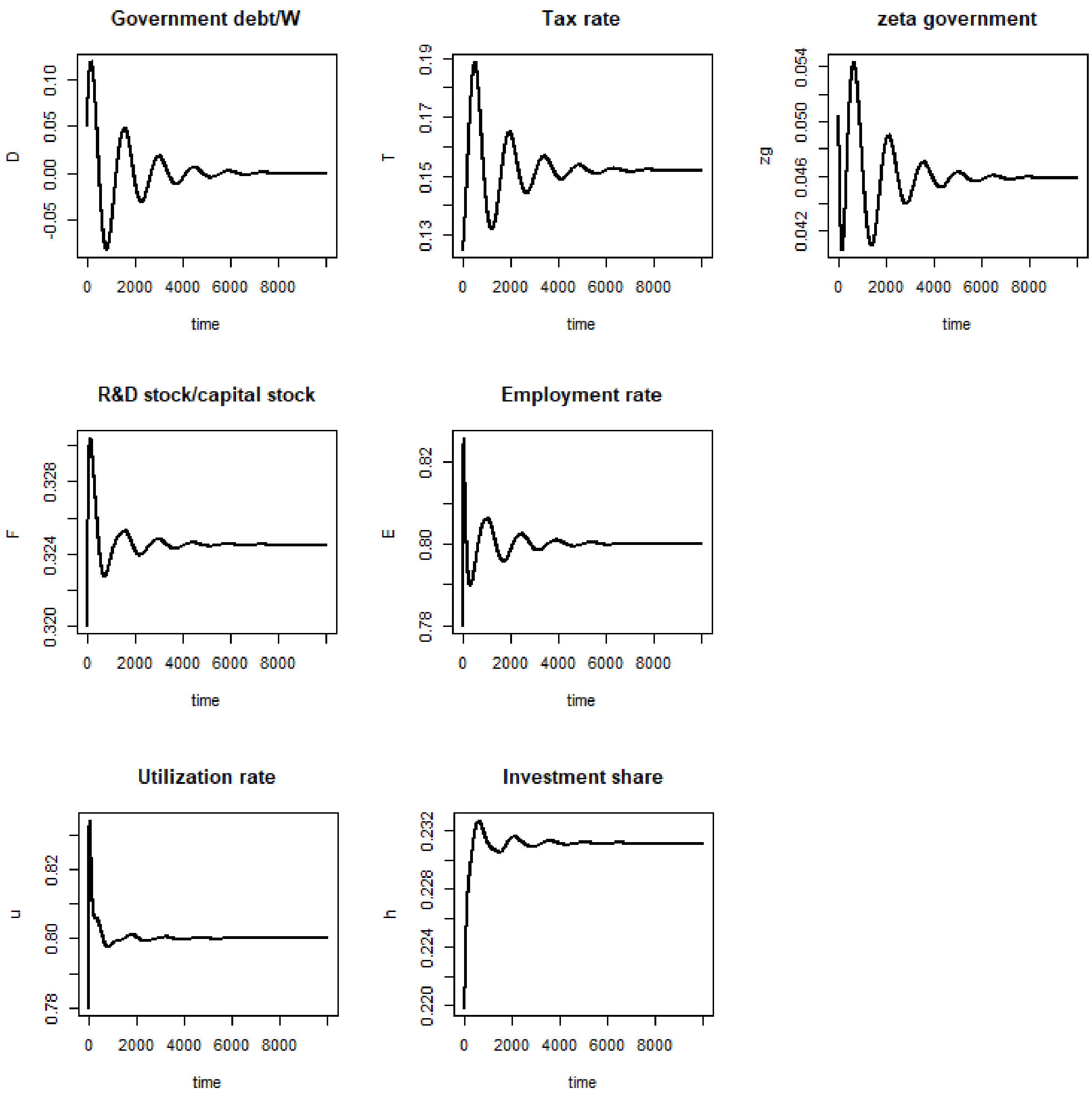


Figure 6. Government model simulation showing stability of the steady state

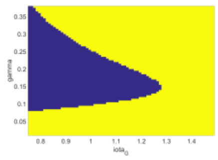
Note that in the set of parameter values that we chose, ι_G is quite a bit larger than η. It makes intuitive sense that this is a condition for government fiscal policy to be an effective stabilizer. Obviously, the primary stabilization mechanism is ζ_G, which adjusts in response to (un)employment (equation 13a). On the other hand, equation (25), which determines the dynamic path of the tax rate, works against such stabilization, because a higher tax rate will decrease private consumption. Thus, any positive effects on GDP and employment from increasing ζ_G will be offset by an increasing tax rate. If the tax effect is immediate, fiscal policy will become ineffective in stabilizing the economy.

The stability of the steady state in the model with a government seems more precarious than the model without a government that was presented above, i.e., government stabilization using fiscal policy is harder than with the private consumption smoothing stabilizer in the previous sections. One of the reasons for this is that whereas before we had two adjustment parameters (γ and ι) for which we needed particular values, we now have three such adjustment parameters: η, γ and ι_G (and we also have $\overline{D}$, which we fixed at zero for mathematical convenience). Figure 7 presents 2D stability diagrams for each combination of two of these parameters. The underlying data for these diagrams is calculated in the same way as for the previous model, i.e., using Matlab's symbolic math module. In the next diagram, we fix the following parameters: $\tau = 0.05$, $\delta = 0.05$, $v = 2$, $\mu = 0.8$, $\bar{E} = 0.8$, $c = 0.6$, $\sigma = 0.7$, $\varphi = 0.1$, $\zeta_h = 0.02$ and $\bar{\rho} = 0.01$.

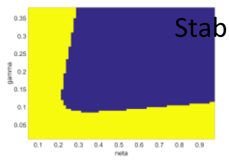
Focusing on γ and ι_G first (top part of the figure), we see that for low values of γ, the model is always stable, irrespective of the value of ι_G, but note that η is fixed at 0.25 (which is a fairly low value) in this diagram. Also high values of γ yield a stable steady state, but intermediate γ values require a high value for ι_G for the steady state to be stable. For the combination η and γ (middle part), we need at least one of these two parameters to have a low value (but note that ι_G is fixed at 0.95, which is a fairly high value). Finally, for the combination of η and ι_G we see that either low or high values of η yield a stable steady state, but for intermediate values of η, we require high values of ι_G (here γ is fixed at 0.1, which is fairly low).

Stable

η is fixed at 0.25



ι_G is fixed at 0.95



Stable

Stable

γ is fixed at 0.1

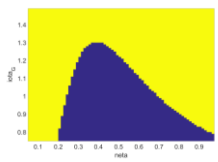


Figure 7. The role of parameters η, ι_G and γ in stability of the steady state in the government model

The rate of technological change and in particular the value of the R&D parameter τ and the private propensity to consume out of wealth (ζ_h) also have an impact on the stability of the steady state. This is shown in Figure 8, which graphs stability in the τ vs. ζ_h plane, and where we fix $\delta = 0.05, v = 2, \mu = 0.8, \bar{E} = 0.8, c = 0.6, \sigma = 0.7, \varphi = 0.1, \eta = 0.1, \iota_G = 0.95, \gamma = 0.01$ and $\bar{\rho} = 0.01$.

Here we see a narrow band of stability emerging, which depicts a tradeoff between the two parameters. High values of one of these parameters require low values of the other for the steady state to be stable, and vice versa. A combination of high ζ_h and high τ also yields negative values for the steady state of T and ζ_G, as can be seen in equations (27) and (28), and which are hard to interpret economically. Thus, the upper-right corner of instability in the figure corresponds to this counter-intuitive situation of negative taxes. The lower-left corner of instability corresponds to an economy with low growth rates (due to low R&D investment) and also low private autonomous spending.

Stable

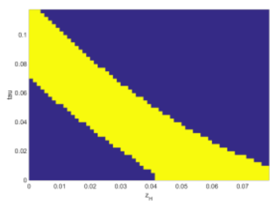


Figure 8. The role of parameters τ and ζ_h in stability of the steady state in the government model

4. Conclusions

The models that we developed in this paper confirm that a long-run steady state with economic growth in a (super)multiplier-based economy with endogenous autonomous consumption spending can be consistent with a stable employment rate. The magnitude of the growth rate is determined by supply-side parameters related to R&D and technical change. But demand is very important in this economy, because without adjustment of demand to the R&D-based growth rate, no stable path exists.

In order to have a stable long run employment rate, autonomous consumption needs to grow at the same rate as labour productivity, something that is not easy to imagine if both these rates are exogenous (as in the basic SSM approach). Our model proposes that the equality of these two rates is obtained by consumption smoothing by wage earners (workers), who adjust their autonomous consumption spending as a fraction of their accumulated savings, in response to unemployment, and/or by government fiscal policy, where the government runs a temporary deficit (surplus) if the unemployment rate is high (low) and raises taxes to keep its long-run debt within bounds.

The (numeric) analysis of the Jacobian matrix of our models has shown that both these stabilization mechanisms (private consumption smoothing and government fiscal policy) will keep the growth path stable, provided that certain parameter restrictions are satisfied. In the case that stabilization takes place by workers' consumption smoothing alone, the ratio of the responsiveness of workers' autonomous spending to unemployment to the responsiveness of firms' investment to capital utilization must be large enough. What this minimum value is exactly depends on parameters in the model, such as the share of wages in GDP, R&D investment as a fraction of GDP, and the rate of autonomous consumption spending by profit earners.

In the model with government fiscal policy as the stabilization mechanism, stability is more difficult to obtain. In this model, various parameters, including the adjustment parameters that govern fiscal policy, but also the responsiveness of firms' investment to capital utilization, as well as the R&D intensity parameter and the propensity of the private sector to consume out of wealth, are crucial parameters that determine stability of the steady state.

Our model variety without a government includes a general consumption function, which allows for consumption spending by workers and by profit owners, and both autonomous consumption (not related to current income) and non-autonomous. Assuming that no consumption (autonomous or otherwise) is done out of profit income simplifies the steady state expressions for the variables in our model, but does not change any of the basic conclusions about growth or stability of the growth path. Also, while most of the time we assume that accumulated workers' savings earn no return, an alternative (and rudimentary) way of modelling such returns suggests that the growth path is unaffected by this (although the distribution of wealth between workers and profit earners is affected). In the model with government fiscal policy, our simplifying assumption is that the government strives for its long-run debt to be zero, and we disregard monetary policy.

In the resulting model, both productivity growth and the growth of autonomous demand indeed appear to be crucial for the emergence of a stable growth path in which we also have stable employment. Technological change (which is modelled by semi-endogenous R&D investments) relieves the resource constraint that the size of the labour force imposes on the economy, and hence makes it possible to achieve per capita growth. Endogenous demand, including endogenous autonomous consumption, keeps the economy on a path where the labour resource is used (at a fixed rate), so that the opportunities provided by technological change are actually utilized.

We feel that there are two main directions in which our model should be extended in future work. On the one hand, while we fully endogenized consumption demand, technological change was only semi-endogenized. Thus, while we considered R&D investment as a fraction of GDP as a fixed parameter, there is scope to consider it as an endogenous variable. This could be done both by making R&D dependent on other macroeconomic variables, such as (expected) profits (as in the endogenous growth literature, e.g., Aghion & Howitt, 1992), or by a behavioral approach that considers R&D at the firm level as resulting from imitation and behavioral mutation (as in Silverberg and Verspagen, 1994). Government spending may also be crucial in the field of technological change (as also in, e.g., in Deleidi & Mazzucato, 2019). While in our current model the rate of growth is ultimately determined by the R&D parameters, with autonomous demand adjusting, fully endogenizing R&D would open the possibility for a model where demand and supply mutually adjust to each other.

On the other hand, the introduction of a more detailed way of modelling the financial sector would also enhance the degree of realism of the model. This would not only allow the modelling of the (de-)stabilizing effects of finance, but also the inclusion of monetary policy by the government.

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Appendix 0. Notation

Variables		*Parameters*	
a	Labour productivity	c_w	Marginal propensity to consume out of wage income
E	Employment rate	c_p	Marginal propensity to consume out of profit income
C	Consumption expenditures	c	Marginal propensity to consume irrespective of income source
D	Government debt as a share of private accumulated savings	$\bar{E}$	Normal employment rate
g	Growth rate of GDP	r	Profit redistribution on the basic of the shares of accumulated savings out of profit or wages
g_K	Growth rate of the capital stock (and of full-capacity GDP)	s	Marginal propensity to save out of current income
g_W	Growth rate of accumulated savings out of wage income	γ	Speed of adjustment of *h* wrt *u*
g_Z	Growth rate of autonomous consumption spending (exogenous in the basic SSM model)	δ	Depreciation rate of the capital stock
G	Total outstanding government debt	ζ_G	Government spending as a share of private accumulated savings
h	Propensity (out of GDP) to invest	ζ_h	Propensity to consume out of accumulated private savings
I	Investment	ζ_p	Propensity to consume out of accumulated savings out of profit
L	Employment	η	Speed of adjustment of tax rate
K	Capital stock	ι_G	Speed of adjustment of share of government spending of private accumulated savings
R	R&D capital stock (accumulated knowledge)	ι	Speed of adjustment of propensity to consume out of accumulated savings out of wages
T	Tax rate	μ	Normal long-run capacity utilization rate
x	Share of accumulated savings out of wage income in total accumulated savings	$\bar{\rho}$	Exogenous part of the growth rate of labour productivity
Y_K	Full-capacity GDP	ν	Normal capital output ratio
Y	GDP		
W_p	Accumulated savings out of profit income	σ	The share of wages in GDP
W_w	Accumulated savings out of wage income	τ	Propensity (out of GDP) to invest in R&D
u	Rate of capacity utilization	φ	Technological opportunities
Z_g	(Autonomous) government spending	Δ	Depreciation rate of the R&D capital stock (usually equal to δ)
Z_h	Autonomous consumption spending by private sector, irrespective of income source		
Z_p	Autonomous consumption spending by profit earners		

	Variables	*Parameters*
Z_w	Autonomous consumption spending by wage earners	
Z	Autonomous consumption spending, unspecified income source	
ρ	Growth rate of labour productivity	
ζ_w	Propensity to consume out of accumulated savings out of wages	
Θ	R&D expenditures	
Φ	Ratio of the R&D-capital stock to the stock of fixed capital	

Appendix 1. The model with generalized consumption equation

Our model with generalized consumption (and savings) behaviour starts from equation (1) in the main text, which is the consumption function:

$$C = \sigma Y c_w + (1-\sigma) Y c_p + Z_w + Z_p \quad (2)$$

Next we define savings as any income that is not consumed, and we distinguish between savings out of labour income and out of profit income. Savings out of labour income are $S_w = \sigma Y - \sigma Y c_w - Z_w$, while savings out of profit income are $S_p = (1-\sigma)Y - (1-\sigma)Y c_p - Z_p$. Note that, by the usual identity, total savings are equal to total investment (R&D and fixed capital):

$$Y = \sigma Y c_w + (1-\sigma) Y c_p + Z_w + Z_p + hY + \tau Y \Rightarrow S_w + S_p = (h+\tau)Y \quad \text{(A1)}$$

We assume that total current investment $(h+\tau)Y$ accumulates into a stock that is held by profit-earners. By slightly re-writing equation (A1) to

$$S_w = (h+\tau)Y - S_p \quad \text{(A1')}$$

we see that savings from labour income are matched by the excess of investment over savings from profit income. Although it is possible that total investment is smaller than savings from profit income, most of our analysis will focus on the case where savings out of labour income are positive, and hence investment exceeds savings out of profit income.

This implies that workers build up positive assets, and that these assets represent holdings on the profit-earning class (firms). Firms, however, also build up assets, which are the means of production (R&D capital and fixed capital), hence workers' assets represent holdings on these means of production. But capital also depreciates, which diminishes the value of the total assets of labour and profit earners together. We choose to attribute depreciation to both workers and profit earners, in proportion to total (net) assets held by each class.[11]

As was done in the main text, we assume (for mathematical convenience) that R&D capital and fixed capital depreciate at the same rate $\delta = \Delta$. This leads to the following equation for the accumulation of assets held by workers, which we denote by W_w:

$$\dot{W}_w = (1 - c_w)\sigma Y - Z_W - \delta W_w \quad \text{(A2a)}$$

The corresponding assets held by profit earners are denoted by W_p, and these accumulate according to

$$\dot{W}_p = \left(1 - c_p\right)(1 - \sigma)Y - Z_p - \delta W_p \quad \text{(A2b)}$$

Now with $Y = (Z_w + Z_P)/(1 - c_w\sigma - c_P(1 - \sigma) - \tau - h)$, equations (A2a) and (A2b) imply

$$\dot{W} = \dot{W}_w + \dot{W}_p = (h + \tau)Y - \delta(W_w + W_p) \quad \text{(A3)}$$

With the initial condition $W(0) = W_w(0) + W_p(0) = R(0) + K(0)$ (the brackets indicate time periods), equation (A3) is guaranteed that $W(t) = R(t) + K(t)$ also for all times $t > 0$. In other words, total assets (wealth) available in the economy is equal to the total amount of production factors (excluding labour).

Admittedly, this means that there is essentially no, or a very limited, role for financial markets in our model. To the extent that financial markets exist, their only role is to channel savings between profit and wage earners, with the ultimate sole aim to fund the expansion of productive capacity. While here and in most of the main text we assume that there is no rate of return on the ("financial") assets held by wage earners, in a further appendix below, we extend the treatment of financial markets to include a rate of return paid to the holders of financial assets (workers). This changes the steady state value for the distribution of wealth (x), but does not affect the growth rate of the economy.

As explained in the main text, our main assumption on autonomous spending is that it depends on wealth, i.e., the variables W_w and W_p. Allowing for autonomous consumption by both wage earners and profit earners, we stipulate

$$Z_w = \zeta_w W_w \quad \text{(A4a)}$$

and

$$Z_p = \zeta_p W_p \quad \text{(A4b)}$$

Note that (A4a) has also been specified in the main text. The ζ parameters are propensities to consume out of wealth for wage earners and profit earners, respectively. With the variable x defined as the share of W_w in total wealth ($W_w + W_p$), we immediately have

[11] Alternative assumptions are possible, but make the mathematics more involved. Generally, making different assumptions about how depreciation of wealth is handled only affects the steady state distribution of wealth (x), not the growth rate.

$$\zeta = \frac{Z}{W} = \zeta_w x + \zeta_p(1-x) \quad \text{(A5)}$$

As specified in the main text, we will assume that ζ_w is a variable, for which we specify a differential equation. On the other hand, ζ_p is assumed to be a constant parameter (and the main text assumes $\zeta_p = 0$).

With the equations specified so far, we are able to write a few of the key growth rates in the model:

$$g_W \equiv \frac{\dot{W}_w+\dot{W}_p}{W} = \frac{(1-c_w)\sigma Y - Z_W - \delta W_w + (1-c_p)(1-\sigma)Y - Z_p - \delta W_p}{W} = \zeta\left(\frac{\tau+h}{1-c_w\sigma-c_P(1-\sigma)-\tau-h}\right) - \delta \quad \text{(A6)}$$

$$g_Z = \frac{\dot{W}}{W} + \frac{\dot{\zeta}}{\zeta} = \zeta\left(\frac{\tau+h}{1-c_w\sigma-c_P(1-\sigma)-\tau-h}\right) - \delta + \frac{\dot{\zeta_w}x+\zeta_w\dot{x}-\zeta_p\dot{x}}{\zeta_w x+\zeta_p(1-x)} \quad \text{(A7)}$$

$$g = g_Z + \frac{\dot{h}}{1-c_w\sigma-c_P(1-\sigma)-\tau-h} \quad \text{(A8)}$$

Remember that g is the growth rate of GDP.

Finally, the differential equation for x can be derived as follows:

$$\dot{x} = \frac{\dot{W_w}}{W_w+W_p} - \frac{W_w(\dot{W_w}+\dot{W_p})}{(W_w+W_p)^2} = \frac{(\zeta_w x+\zeta_p(1-x))((1-c_w)\sigma-x(\tau+h))}{1-c_w\sigma-c_P(1-\sigma)-\tau-h} - \zeta_w x \quad \text{(A9)}$$

We can also summarize the other five (in addition to A9) differential equations that make up the model with the generalized consumption function (equation numbers refer to the main text, and remember $\Delta = \delta$):

$$\dot{h} = h\gamma(u-\mu) \quad \text{(1)}$$

$$\dot{\Phi} = \Phi\frac{u}{v}\left(\frac{\tau}{\Phi} - h\right) \quad \text{(11)}$$

$$\dot{\zeta_w} = \iota\zeta_w(\bar{E} - E) \quad \text{(13)}$$

$$\dot{u} = u\left(\frac{(\zeta_w x+\zeta_p(1-x))(\tau+h)+\dot{h}}{1-c_w\sigma-c_P(1-\sigma)-\tau-h} + \frac{\dot{\zeta_w}x+\zeta_w\dot{x}-\zeta_p\dot{x}}{\zeta_w x+\zeta_p(1-x)} - \frac{hu}{v}\right) \quad \text{(A10)}$$

$$\dot{E} = E\left(\frac{(\zeta_w x+\zeta_p(1-x))(\tau+h)+\dot{h}}{1-c_w\sigma-c_P(1-\sigma)-\tau-h} + \frac{\dot{\zeta_w}x+\zeta_w\dot{x}-\zeta_p\dot{x}}{\zeta_w x+\zeta_p(1-x)} - \delta - \bar{\rho} - \varphi\Phi\right) \quad \text{(A11)}$$

These equations can be solved for the steady state of the model using the procedure outlined in the main text. The steady state expressions presented in the main text are specific for the assumptions $c_P = 0$ and $\zeta_p = 0$. The generalized steady state expressions for u, E, Φ and h are identical to the expressions in the main text. The generalized steady state expressions for the two remaining variables are

$$x^* = \frac{v(1+\Phi^*)(\zeta_p+\delta+\bar{\rho}+\varphi\Phi^*) - \mu s_P(1-\sigma)}{v(1+\Phi^*)(\zeta_p+\delta+\bar{\rho}+\varphi\Phi^*)} \quad \text{(A12)}$$

$$\zeta_w^* = \frac{[\mu(\sigma s_w+(1-\sigma)s_P) - v(\delta+\bar{\rho}+\varphi\Phi)(1+\Phi^*)](\zeta_p+\delta+\bar{\rho}+\varphi\Phi^*) - \zeta_p\mu s_P(1-\sigma)}{v(1+\Phi^*)(\zeta_p+\delta+\bar{\rho}+\varphi\Phi^*) - \mu s_P(1-\sigma)} \quad \text{(A13)}$$

In these expressions, we substituted the definitions $s_P \equiv 1 - c_P$ and $s_w \equiv 1 - c_w$.

Appendix 2. Introducing a rate of return to workers' assets

As explained in the main text, we model the rate of return on accumulated workers' savings by the introduction of a parameter r, which represents the fraction of profits that is paid to workers in return for their savings, which are used by firms (profit earners) to pay for the investments in fixed capital and R&D. With the inclusion r into the model, workers will receive a share $\sigma + rx(1-\sigma)$ of GDP, and profits earners a share $(1-\sigma)(1-rx)$. This changes the equations (equations A2a and A2b) for accumulation of wealth:

$$\dot{W}_w = s_w(\sigma + rx(1-\sigma))Y - \zeta_w W_W - \delta W_w \quad \text{(A2a')}$$

$$\dot{W}_p = s_p(1-\sigma)(1-rx)Y - \zeta_p W_p - \delta W_p \quad \text{(A2b')}$$

It also changes the multiplier, as can be seen in the equation for output:

$$Y = Z_w + Z_p + c_w\big(\sigma + rx(1-\sigma)\big)Y + c_p\big((1-\sigma)(1-rx)\big)Y + (h+\tau)Y \Rightarrow$$

$$Y = \frac{Z_w + Z_p}{1 - c_w\big(\sigma + rx(1-\sigma)\big) - c_p\big((1-\sigma)(1-rx)\big) - (h+\tau)} \quad \text{(2a')}$$

Note that in this equation, we have assumed that the marginal propensity to consume out of current income is unchanged, both for workers and profit earners, even if with $r > 0$, workers' income is partly profits (or returns on savings).

From these basic changes associated to the introduction of r, the differential equations of the model can be derived in the same way as before. We find that three equations change, specifically:

$$\dot{u} = u\left(\frac{\big(\zeta_w x + \zeta_p(1-x)\big)(h+\tau) + \dot{h} - \dot{x}(1-\sigma)(s_p - s_w)}{1-(1-s_w)(\sigma + rx(1-\sigma)) - \big(1-s_p\big)(1-rx)(1-\sigma) - (h+\tau)} + \frac{\dot{\zeta_w}x + \zeta_w\dot{x} - \zeta_p\dot{x}}{\zeta_w x + \zeta_p(1-x)} - \frac{hu}{v}\right) \quad \text{(A10')}$$

$$\dot{E} = E\left(\frac{\big(\zeta_w x + \zeta_p(1-x)\big)(h+\tau) + \dot{h} - \dot{x}(1-\sigma)(s_p - s_w)}{1-(1-s_w)(\sigma + rx(1-\sigma)) - \big(1-s_p\big)(1-rx)(1-\sigma) - (h+\tau)} + \frac{\dot{\zeta_w}x + \zeta_w\dot{x} - \zeta_p\dot{x}}{\zeta_w x + \zeta_p(1-x)} - \delta - \bar{\rho} - \varphi\Phi\right) \quad \text{(A11')}$$

$$\dot{x} = \frac{\big(\zeta_w x + \zeta_p(1-x)\big)[s_w(\sigma + rx(1-\sigma)) - x(h+\tau)]}{1-(1-s_w)(\sigma + rx(1-\sigma)) - \big(1-s_p\big)(1-rx)(1-\sigma) - (h+\tau)} - \zeta_w x \quad \text{(A9')}$$

This model (which also includes equations 4, 11 and 13) can be solved for the steady state in essentially the same way as has been done for the case $r = 0$. Related to the fact that in equations (A10') and (A11'), the parameter r only appears in the multiplier (as in A3a'), the steady state solutions for h and Φ do not change. With equations (4) and (13) unchanged, the steady state solutions for u and E also do not change. Thus, we obtain only steady state expressions for x^* and ζ_w^* that are different than before, while the other steady state expressions do not change:

$$x^* = \frac{\frac{u}{v}s_p(1-\sigma) - (1+\Phi^*)\big(\zeta_p + \delta + \bar{\rho} + \varphi\Phi^*\big)}{\frac{u}{v}s_p(1-\sigma)r - (1+\Phi^*)\big(\zeta_p + \delta + \bar{\rho} + \varphi\Phi^*\big)} \quad \text{(A14)}$$

$$\zeta_w^* = \frac{\mu}{v}\frac{(s_w - s_p)r(1-\sigma)}{(1+\Phi^*)} + \frac{\frac{\mu}{v}\big(s_w\sigma + s_p(1-\sigma)\big) - (1+\Phi^*)(\delta + \bar{\rho} + \varphi\Phi^*)}{(1+\Phi^*)}\frac{1}{x^*} - \zeta_p\frac{\frac{\mu}{v}s_p(1-\sigma)(1-r)}{\frac{\mu}{v}s_p(1-\sigma) - (1+\Phi)\big(\zeta_p + \delta + \bar{\rho} + \varphi\Phi\big)} \quad \text{(A15)}$$

Especially the expression for ζ_w^* is rather complicated. However, it can easily be seen that with $r = 0$ it reduces to equation (A13). Similarly, equation (A14) reduces to (A12) for the case $r = 0$. We can also reduce these steady state expressions for the case $r = 1$:

$$x^* = 1 \quad \text{(A16)}$$

$$\zeta_w^* = \frac{\mu}{v(1+\Phi^*)} s_W - (\delta + \bar{\rho} + \varphi\Phi^*) \quad \text{(A17)}$$

Appendix 3. Details of the government stabilization model

There are a number of equations that change slightly in the model with a government sector. This appendix presents the details of these equations. First, the introduction of taxes implies that consumption is a function of disposable income:

$$C = Z_H + c(1 - T + (1 - \sigma)D)Y \quad \text{(1a)}$$

Using also the definitions that are specified in the main text, this leads to the following equation for GDP:

$$Y = (Z_H + Z_G)\frac{1}{1-c(1-T+(1-\sigma)D)-(h+\tau)} \quad \text{(2b)}$$

Because government bonds are held by private agents (any increase of G will correspond to a private surplus), total assets held by private agents are equal to $W + G$. This modifies the equation for private autonomous spending to

$$Z_h = \zeta_h(W + G) \quad \text{(A4c)}$$

The equation for government autonomous spending is specified in the main text (A4d), and together these two equations lead to

$$Z = Z_G + Z_h = (\zeta_G + \zeta_h)W + \zeta_h G \quad \text{(A4e)}$$

In line with our previous derivations (Appendix 1), we also have

$$\frac{\dot{W}}{W} = \frac{(h+\tau)\,[(\zeta_H+\zeta_G)+\zeta_H D]}{1-c(1-t+(1-\sigma)D)-(h+\tau)} - \delta \quad \text{(A3b)}$$

And from equations (23) and (1a) as well as the definition of D, it follows that

$$\frac{\dot{G}}{G} = \frac{1}{D}\left[\zeta_G + \frac{((1-\sigma)D-t)[(\zeta_H+\zeta_G)+\zeta_H D]}{1-c(1-t+(1-\sigma)D)-(h+\tau)}\right] \quad \text{(A18)}$$

Then it can easily be seen that

$$g_Z = \frac{\dot{\zeta}_g-(\zeta_H+\zeta_G)\delta+\zeta_H\zeta_G}{(\zeta_H+\zeta_G)+\zeta_H D} + \frac{(\zeta_H+\zeta_G)(h+\tau)+\zeta_H((1-\sigma)D-t)}{1-c(1-t+(1-\sigma)D)-(h+\tau)} \quad \text{(A7a)}$$

With this new equation for g_Z, we also have new equations for $\dot{u}$ and $\dot{E}$:

$$\dot{u} = u\left(\frac{\dot{\zeta}_g-(\zeta_H+\zeta_G)\delta+\zeta_H\zeta_G}{(\zeta_H+\zeta_G)+\zeta_H D} + \frac{(\zeta_H+\zeta_G)(h+\tau)+\zeta_H((1-\sigma)D-t)-c\dot{t}+c(1-\sigma)\dot{D}+\dot{h}}{1-c(1-t+(1-\sigma)D)-(h+\tau)} - \frac{hu}{v} + \delta\right) \quad \text{(7a)}$$

$$\dot{E} = E\left(\frac{\dot{\zeta}_g-(\zeta_H+\zeta_G)\delta+\zeta_H\zeta_G}{(\zeta_H+\zeta_G)+\zeta_H D} + \frac{(\zeta_H+\zeta_G)(h+\tau)+\zeta_H((1-\sigma)D-t)-c\dot{t}+c(1-\sigma)\dot{D}+\dot{h}}{1-c(1-t+(1-\sigma)D)-(h+\tau)} - \bar{\rho} - \varphi\Phi\right) \quad \text{(8a)}$$