# A Tolerance Management Domain Model (ToleranceDM) for Semantic Enrichment of BIMs

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**Abstract:** Dimensional variability of components and assemblies in construction can lead to significant defects, rework, and project risk if not managed effectively. Given the complexity of using tolerance management to control dimensional variability, an automated BIM-based approach is highly propitious, while currently elusive. This paper develops the first iteration of a domain model for tolerance management (ToleranceDM) using two case study examples within the domain of building construction. The results are shown to (1) consolidate the scattered, disparate existing "knowledge" and research on tolerance management into a single standardised, uniform framework, and (2) formalise this knowledge so that it can be unambiguously interpreted and parsed into software systems for automated tolerance management in construction. ToleranceDM functions as a key step towards benchmarking process capabilities, computing tolerance compliance automatically, and enabling in-field communication of tolerance requirements. Future research should explore case studies in different construction domains, along with developing an improved abduction framework and integrating as-built project data for tolerance compliance checking.

**Keywords:** domain model, dependency structure matrix, tolerance management, building information model, tolerance analysis, risk management

## 1.0 Introduction

The impacts of dimensional variability are well documented in literature, yet continue to be a source of rework, cost-overruns, delays, and litigation on construction projects. Frequent examples reveal how poor project performance can be directly tied to improper consideration and control of dimensional variability [1-4]. In severe cases, these impacts can completely halt projects and be the single largest contributor to cost and time overruns [5]. Tolerance management has emerged within the construction industry as a dependable practice for ensuring dimensional variability is controlled within allowable limits [6-8]. State-of-the-art for tolerance management in construction is still manual and tedious, eliciting an extensive understanding of theory that is perceived as difficult to implement in practice [9]. As a result, it can sometimes be deemed more cost-effective to not allocate any resources or consideration for proper tolerance management and opt to solve dimensional conflicts reactively in-situ as and when they occur [10]. However, this can significantly increase a project's risk exposure [11].

Tolerance management is focused on employing a set of methods [12] to (1) minimise defects associated with dimensional and geometric variability (called tolerance issues hereafter) in a prescient manner [11,13], and (2) to ensure constructability of components and assemblies [3]. The ultimate aim of tolerance management is to minimise resources spent to modify tolerance issues and ensure structural safety, constructability, aesthetics, and functionality of buildings. These can only be achieved if more judicious decisions are made upstream in the process rather than downstream once construction activities have commenced [6]. Despite its potential value, tolerance management is currently beset by taxing manual activities, which are not conducive for use on most construction projects.

Automated tolerance management has the potential to reduce barriers for application in practice, can be applied proactively, and builds the foundation for advanced tolerance optimisation methods. Motivation for automated tolerance management is evident when considering the requirements of emerging technologies and processes in construction. The proliferation of a fully autonomous or robotic construction site requires codified design rules and assembly processes that cannot be based on manual methods for resolving dimensional variability conflicts (i.e., trial and error or rules of thumb practices) [14]. For instance, large-scale additive manufacturing is highly time-sensitive and the final geometry is based on complex design and process parameters [15]. For such processes, researchers have recently demonstrated the efficacy of applying tolerance management principles for characterizing, analysing and controlling the adverse effects of dimensional variability in a prescient manner [16]. Saidi et al. [17] also argue that the state of robotics in construction faces an uphill battle due, in part, to the absence of proper tolerance management. Given the necessity for robotic systems to rely on and employ digital information, an automated tolerance management system is essential for further advancing the state of automation in construction.

In order to deploy automated tolerance management in construction, it is first necessary to have semantic information enriched within a Building Information Model (BIM) that contains necessary information about tolerances. Since a generalisable process for such semantic enrichment is currently elusive, this paper presents a novel tolerance domain model (i.e., a structured collection of tolerance concepts and relationships between those concepts, similar to the notion of a "schema", or ontology) for use in construction. This model, herein referred to as ToleranceDM, collects, classifies, and evaluates information from the academic literature, codes of practice, design standards (specifically for steel and concrete), and expert knowledge.

The basic function of ToleranceDM is to take an initial BIM and to enrich relevant tolerance information into it using pre-defined rules, a design structure matrix for describing connection points and an inference engine for integrating risk. In doing so, ToleranceDM addresses three prevailing challenges facing automated tolerance management in construction. First, it serves as a centralized mechanism for collecting disparate tolerance management knowledge (such generated knowledge is currently siloed across projects). Second, it functions as a generalizable framework that can be applied across a broad range of construction projects. Third, it explores the required Level of Development (LOD) in a BIM for initiating various tolerance management tasks (e.g., tolerance analysis). LOD is a specification schema that prescribes generally agreed-upon levels of development of a BIM. It is intended to be used to improve the communication of 3D information contained within a BIM. Within the context of prescribing tolerances, the building information modelling LOD is particularly important since each LOD (i.e., there are six general levels: LOD 100, LOD 200, LOD 300, LOD 350, LOD 400, and LOD 500) prescribes distinct subtleties in terms of object features, geometric envelope accuracy, internal sub-components, and connection detail fidelity [18]. For clarity, this paper refers to the LOD specification framework developed by BIMForum [19], which is initially based on the schema developed by the American Institute of Architects.

The presented approach of ToleranceDM in this paper is shown to scale to real-world, large-scale BIMs. In addition, the range of query features in ToleranceDM provide new critical information about tolerance sensitivity between specific components that supports construction planning. The direct beneficiaries for ToleranceDM will be practitioners whose work centres on the modelling of assembly connections, simulation of construction sequencing, and risk managers who need to make informed decisions about product and process design in construction. Once operational, ToleranceDM will also serve as the foundation for automated tolerance management, unlocking new frontiers for autonomous and robotic construction processes.

This paper is structured as follows. Section 2 provides background information on the current state of tolerance management in construction. Section 3 provides the methodology used for developing ToleranceDM. Section 4 details the step-by-step implementation of this methodology and Section 5 carries out a functional demonstration on two projects (a small simple structure and a large, complex commercial building). Finally, Section 6 provides conclusions and next steps for research.

## 2.0 Background

Ontologies used to describe concepts in tolerance management in construction can be verbose; stemming largely from the dense body of knowledge used for product tolerancing in manufacturing, and from separate attempts in recent decades to apply it to construction [5-7,12]. For clarity, this paper presents a condensed overview of tolerance management, along with key definitions, which are elaborated upon in the following sections.

### 2.1 Related Work on Tolerance Ontologies in Mechanical Product Design

Numerous research efforts have developed ontological approaches for representing and reasoning about tolerances in the field of mechanical product design. What has not been undertaken, to the best of our knowledge, is an ontological account of tolerances in the context of construction. It is useful and important to draw analogies between the two domains in an effort to reuse and adapt knowledge capture approaches, although the kinds of knowledge, the impacts of tolerance, and how tolerance information can be used for decision support is significantly different.

Most ontology-based approaches in product design are used (at various design stages) for partially or fully specifying tolerances, allocating numerical tolerance values (e.g., [20-23]), and for facilitating interoperability between design tools (e.g., [24,25]). In [21] the authors use case-based reasoning to automate the specification of tolerances to mechanical parts in a design, and to automatically allocate (numerical) tolerance values. In [22] the authors use ontological reasoning to automatically specify tolerances to parts on a mechanical design at a very early design stage – notably even before geometric details have been committed to.

In mechanical assembly design, considerable attention is given to the variety of prototype shapes that products take (e.g., surfaces being spherical, cylindrical, planar, helical, revolute, prismatic) and the implications in terms of positional tolerances, how surfaces can have contact, etc. [20]. In construction, there is far less variety in the shape categories of products, and less variety in the ways that typical products have contact. Indeed, for most of the operational lifetime of various products, two products having "contact" does not directly imply physical contact, as products bend, expand, contract, and stretch. Simply knowing that the positioning of two products is highly sensitive can greatly assist during the construction of a building, i.e., special care can then be taken to ensure the proper placement of the two products, as specific kinds of deviation may have costly ramifications at a later stage in construction.

### 2.2 Variations and Tolerances

Components in construction are specified with geometric dimensions and key material properties. While nominal dimensions are communicated during design, when components and assemblies are constructed, dimensions cannot be achieved with 100% accuracy. The discrepancy or deviation between the nominal and actual dimension is defined as a dimensional variation [26,27]. The acceptable amount of this variability is defined as the tolerance of a geometric entity (e.g., width, length, height, etc.) or installed position of a component [8,28,29]. In some cases, a distinction is made between a dimensional tolerance (limit placed on size), and geometric tolerance (limit placed on form such as straightness or waviness). Within product tolerancing, variations and tolerances are distinguished between features of an object (e.g., surface, edge, or profile), or the relationship between objects within an assembly. For clarity, this paper consolidates these aspects of tolerance management as intra-object tolerances (i.e., within or related to one object) and inter-object tolerances (i.e., between or related to multiple objects in an assembly). These two aspects are known as tolerance sensitive relations hereafter.

Dimensional and geometric variability is the result of many interacting factors; material-related and process-related [30-32]. During design, it is important to ensure that components are properly "toleranced", such that when aggregated, they can fit together properly. The design must also account for the unique types of construction methods employed (e.g., variations range from less than a millimetre for many factory-made components to several centimetres for many in-situ components) [8,33]. The physical aggregation process also induces geometric variations since gravity loads of the building gradually increase during the construction process, potentially resulting in building settlement, movement, and deformation [34].

### 2.3 Risks Associated with Dimensional and Geometric Variations

In light of the intricacies of dimensional and geometric variability, Talebi et al. [35] suggest that effective tolerance management should manage key risks that occur in connections comprising two or more components. Rausch et al. [36] summarize these key risks as being related to structural safety, constructability, aesthetics, and functionality.

*Structural safety risk* relates to dimensional variations that change the load resistance or stability of a structure and examples include column eccentricity, misaligned connections, and mispositioned rebar. *Constructability risk* relates to the dimensional variations which affect the aggregation quality between mating parts and assemblies. At the part level, there are two extreme cases of poor constructability resulting from dimensional variability: (1) a part is too large to connect into its intended interface(s), and (2) a part is too small to connect properly into its intended interface(s). *Aesthetics risk* relates to dimensional variations that impact the perceived quality of a completed assembly, and often occurs from visible misalignments between connecting parts and components. Finally, *functionality risk* relates to dimensional variations that impact the intended performance or serviceability of an assembly. This is best observed in structural design, where a strict limit is placed on floor deflections. While certain deflections may not negatively impact the structural integrity of a floor, serviceability limit states ensure that occupants do not feel unsafe.

### 2.4 State of Tolerance Management in Construction

The state of tolerance management has been traditionally beset by reactive practices. In recent years however, several proactive methods have been developed. Penalty-incentive schemes across trades and optimal arrangement of interchangeable components can reduce tolerance-based risk [2]. Such schemes provide limited efficacy however, since systemic interaction of geometry between components cannot purely be solved during production and aggregation of components. A more efficacious approach is needed to perform mitigation during design and to subsequently communicate tolerances effectively. Several comprehensive design approaches have been previously proposed including strict-vs-loose tolerance allocation strategies, kinematic chain-based dimensional variation analysis and Monte Carlo simulation [10,13,37]. Other studies [6,9,28] have attempted to develop processes by which tolerance management methods can be applied in a consolidated and systematic manner. For example, Talebi [12] proposed the

Tolerance Management System (TMS) by which already developed methods are divided into five categories, namely identification of tolerance requirements and risks, planning the achievement of tolerance requirements and risks, communication of tolerance information, tolerance compliance method, and learning and documentation. In TMS, the identification of critical connections is of prime importance. However, this approach lacks the implementation in the real world to evaluate its practicality. Also, its full implementation is onerous without automation as it contains many manual tasks.

Unfortunately, effective tolerance management is still elusive in many spheres of construction due to the practical barriers facing implementation of existing methods. Often, solutions predicated on manual processes are developed for very specific project demands. As such, there is a need for a holistic process that can be universally applied and that does not warrant extensive background knowledge as required in existing approaches. This is particularly where the use of BIM can afford such a process to be both holistic (as the use of BIM becomes ubiquitous in construction) and automated (as emerging technologies are increasingly digitized). Tolerance management is most effective when applied proactively during the design stage. This paper exploits such an approach using a domain model that can be universally applied and in a highly automated manner.

## 3.0 Methodology for developing the Tolerance Management Domain Model (ToleranceDM)

This research study develops a domain model (DM) for tolerance management. The IDEF5 methodology for knowledge engineering and ontology development [38] is used for the development of the ToleranceDM in this research study. It is emphasized that while IDEF5 knowledge capture *development process* is adopted, EXPRESS is used as the data model specification language in line with industry standards such as the Industry Foundation Classes. IDEF5 is organized into five stages of domain model development: (1) organization and scoping, (2) data collection, (3) data analysis, (4) initial domain model development, and (5) refinement and validation. The following subsections elaborate on how these stages have been undertaken.

### 3.1 Organization and Scoping

In this study, the scope of the DM is specifically directed towards new building construction projects, as demonstrated by the presented case study. However, ToleranceDM is general in nature and has the potential to be readily adapted to any kind of construction project. The long-term agenda of this work is the development of a meta-model that captures a more abstract structure of tolerance concepts that are common across a broader scope of construction domains (e.g., bridges, tunnels, geotechnical fields, etc.). The core pre-requisite for implementing this method is an initial BIM with sufficient LOD at key connection points between components, where tolerance relations are prone to certain types of project risk (e.g., structural safety, constructability, etc.). As such, ToleranceDM is agnostic to aggregation between strictly new construction or between new and in-situ construction (e.g., prefabricated components being used in a building renovation/adaptation project – such projects are often prone to tolerance issues due to varying levels of dimensional variability which must be suitably mediated). For the purpose of demonstrating the initial architecture of ToleranceDM however, this paper focuses specifically on new building construction projects.

### 3.2 Data Collection

The authors of this paper have collectively undertaken seven case studies (previously), in which manifold types of tolerance issues were encountered for the following projects: an industrial pipe rack assembly [13], two 805 $m^2$ modular data centres [36], a small-scale steel bridge [39], a 7,500 $m^2$ commercial building [40], a 2.3 ha terraced warehouse [40], and a prefabricated accessory dwelling unit [41]. These case studies provide valuable tolerance management related information pertaining to a mixed set of project delivery methods (i.e., prefabricated versus in-situ), assembly size, complexity, location, and typology (i.e., commercial, industrial, residential, infrastructure). The tolerance issues identified in these case studies also represent a broad range of defects associated with dimensional variability, including misalignment between structural components, aesthetically unacceptable gaps between structural and non-structural components, and lack of fit in structural frames.

Data for developing ToleranceDM is based on these previous case studies, as well as information extracted from other existing journal articles for tolerance management in construction [3,6,9]. As such, ToleranceDM is built upon existing literature that covers concepts from four domains, namely: architecture, engineering, manufacturing, and construction project management. The purpose of the literature review was to identify the underlying concepts and principles of tolerance management, and to identify the relevant state-of-the-art solutions to automate tolerance management.

Tolerance issues identified in the case studies were categorised based on three areas with high tolerance risks, namely (a) the connection between the building structure and the building envelope, (b) the connection between the building structure and internal components, and (c) where stringent tolerances should be specified in the internal area of the building, as suggested by (REF). This categorisation helped authors gain a better understanding of type of tolerance issues in each specific area. Afterwards, the identified solutions in the literature were divided into four areas: (a) identification of tolerance requirements/risks, (b) planning the achievement of tolerance requirements/mitigating tolerance risks, (c) communication of tolerance information, and (d) tolerance compliance control, as proposed by (REF). In this paper, solutions addressing identifying tolerance requirements/risks as well as communication of tolerance information were considered when developing ToleranceDM.

### 3.3 Data Analysis

After collecting initial information regarding tolerance management, the authors began to analyse and structure the core principles into a cohesive framework. Two types of criteria proposed by Talebi et al. [42] and Hong and Chang [43] were initially found to categorise the overall framework developed in this research. The terminologies needed to create the framework were categorised based on the concepts of *Geometric Dimensioning and Tolerancing (GD&T)* and the tolerance risks proposed by Rausch et al. [36]. GD&T is a symbolic ontology [29] that communicates permitted deviations in form, size, orientation, and location of features (e.g., size or surface) for a component [44]. GD&T is also used to define relationships between components within an overall product (i.e., assembly), and has become the de-facto ontology for dimensional specification within the manufacturing industry [45]. The application of GD&T in construction has been recently demonstrated by Talebi et al. [46], which consists of a condensed set of principles more suited to construction processes.

While analysing data for the proposed framework, three industry experts on tolerance management were consulted with using a set of semi-structured interviews. These industry experts have the following backgrounds: Expert A is an internationally renowned building information modelling consulting expert with experience in design, standards development, and technology management, Expert B is a quality control manager with experience in inspection of assemblies in offsite construction, and Expert C a lead welding inspector with a background on industrial, commercial, and prefabricated steel assemblies. Collectively, these experts provide a rich set of experiences on the design, inspection of, and implementation of tolerance management practices in construction projects across Europe and North America, predominately within large AEC firms. Two semi-structured interviews were held with each expert (during the development process); each ranging in length between 30 and 60 minutes. Through this consultation, the following criteria were selected for the framework: identification of tolerance risks, specification of tolerance values, tolerance analysis, tolerance allocation, and communication of tolerance information. These criteria were selected using external expertise to ensure the developed domain model remained pragmatic and relevant to current industry practice.

### 3.4 Model Development Process

ToleranceDM was developed incrementally using refinement cycles consisting of concept brainstorming, mapping, and organization into inheritance hierarchies, and then reviewing against a body of case studies. The development team (i.e., the authors) serve as five multi-disciplinary experts with the following backgrounds: two with a background in knowledge engineering and artificial intelligence for AEC, two experts on tolerance management in construction with backgrounds in dimensional inspection consulting, and one with a background in simulation, expert elicitation systems and construction innovation. As a key driving motivation towards automated tolerance management in construction, the authors have developed proof of concept software tools for tolerance management in parallel with the development of the DM.

With respect to scoping, in the first version of ToleranceDM, mitigation is only incorporated into the meta-model (i.e., introduced as concepts at the highest level of abstraction). As such, mitigation concepts have not been as fully developed as other aspects of ToleranceDM, as reflected in their superficial treatment in the description of ToleranceDM workflow processes (Section 4) and the integration of ToleranceDM into the IFC standard. In future iterative refinement cycles these aspects will be dealt with in significantly more depth.

### 3.5 Refinement and Validation

The initial validation process consisted of collecting feedback from three external experts using semi-structured interviews. Prototype analysis tools are validated on BIMs from real buildings (rather than purely conceptual models) to ensure that the DM is practical and applicable to real-world construction projects. This drives the development of the DM with the inclusion of concepts that are determined to be valuable or necessary to undertake the analysis tasks required by the cases, and helps to refine the DM by ensuring that redundant or unused concepts can be highlighted and removed, ensuring that the DM is lean and pragmatic. As a result of the feedback and instantiation of DM classes through real projects, the model was refined and queried to ensure responses were both valid and adequate.

## 4.0 Tolerance Management Domain Model (ToleranceDM)

As outlined in Figure 1, the proposed workflow for ToleranceDM takes an initial BIM and through a sequence of inference rules based on expert knowledge about identifying, assessing, and mitigating tolerance-related risks, automatically parses key tolerance relationships to create a semantically rich BIM with tolerance management concepts such as tolerance categories and mitigation strategies. The enriched BIM can then be used to perform a series of useful analyses and simulations to conduct and support effective tolerance management. For clarity, the scope of this present research is on the development of the DM and practical direct use-cases, which is essential to move towards automated tolerance management and can be considered as a starting point for similar research efforts. A more comprehensive examination and demonstration of potential use-cases of BIMs enriched through ToleranceDM will be addressed in future work.

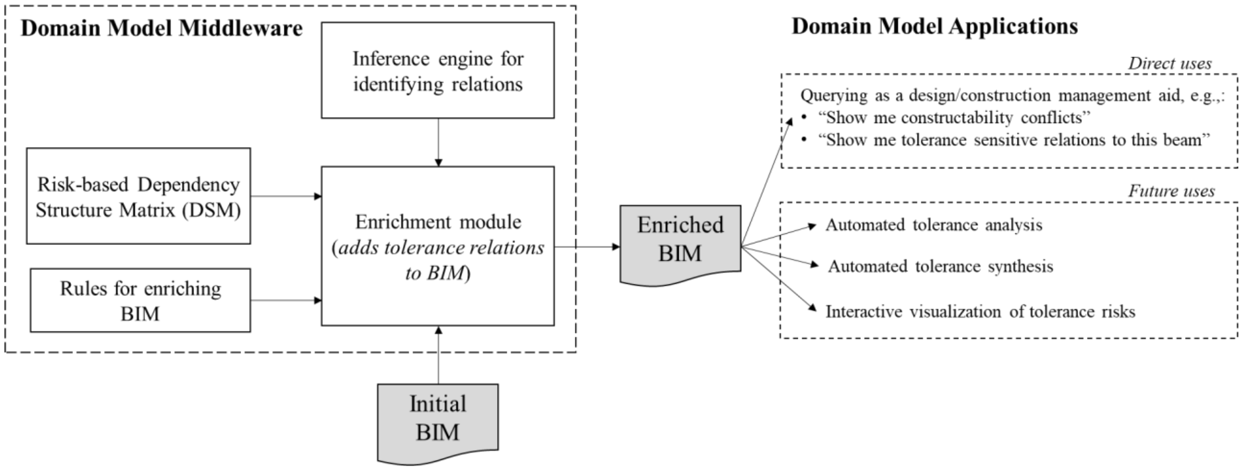


***Figure 1: Overall workflow for the Tolerance Management Domain Model (ToleranceDM)***

More concretely, the current workflow utilising ToleranceDM is divided into two processes, each illustrated as Business Process Model Notation diagrams in Figure 2 and Figure 3. The BPMN swim lanes (large, rounded marquee rectangles) denote the division of roles between different actors, and BPMN activities (small, rounded rectangle boxes) and artefacts (dog-eared boxes) detail which knowledge is available at each stage in the two processes.

The first process (Figure 2) is undertaken by BIM engineers at an organisational (company) level. Initially the organisation develops their own custom, tailored Risk-Based Dependency Structure Matrix (DSM) that expresses the tolerances specific to the projects that they are involved in. We envision in the future that this step will be significantly supported through a community-wide effort in developing open libraries of template DSMs that capture domain specific knowledge about common tolerance sensitive relations between particular building features. ToleranceDM provides the modelling language "building blocks" that describes the classes of tolerance sensitivity, the current version of which we present in Section 4, especially Table 1, Figure 4 and Figure 6.

Next, BIM engineers specify a mapping between BIM products and their in-house custom product types. That is, we determined through our case study of the Oastler building that many critical tolerance relationships exist between product types that did not align well with the BIM data exchange format that was used (e.g., slabs that formed parts of the foundation, and a particular architectural feature referred to as "fins" in the Oastler building). Thus, from the perspective of ToleranceDM, rather than tying the identification of products in the DSM to a particular BIM standard (e.g., IFC), the workflow introduces an additional abstraction layer in which an organisation "maps" BIM products into their own custom types. In principle, this is an optional step and an organisation may opt for a 1-to-1 relationship between product types in their DSM and the underlying BIM standard. The DSM and product mapping artefacts are subsequently iteratively refined during and between projects.

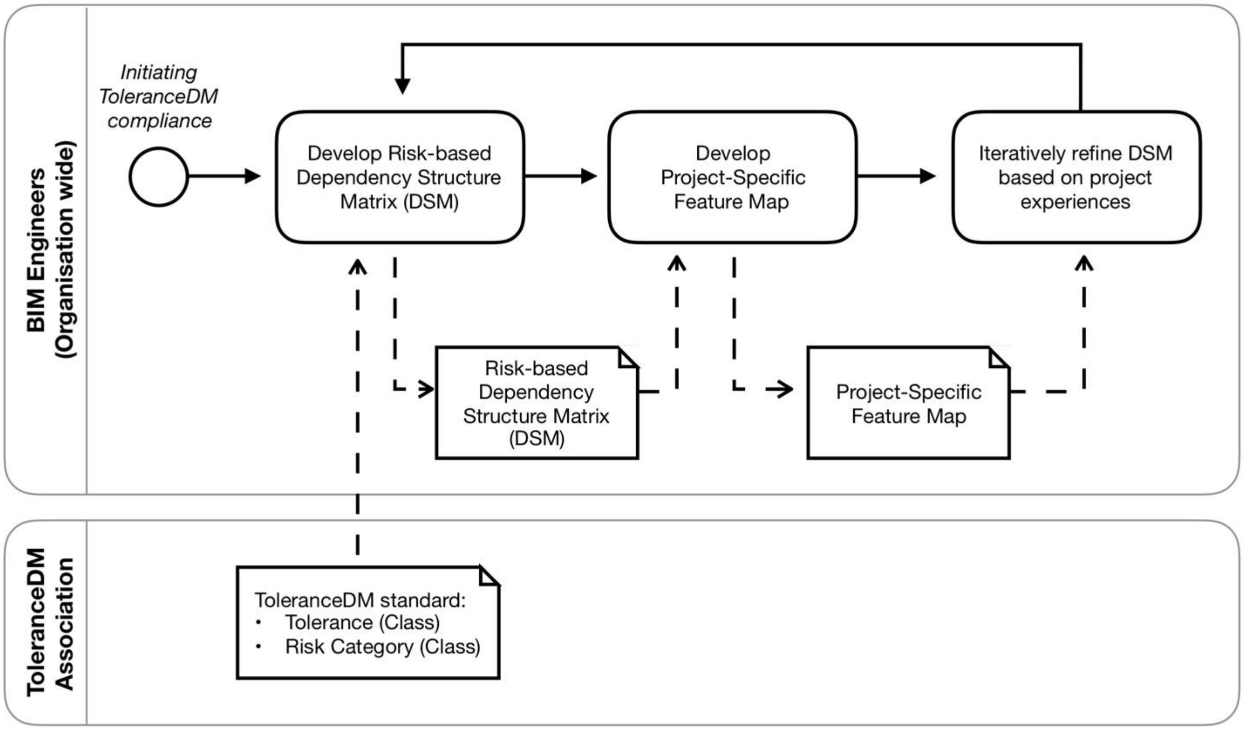


***Figure 2: BPMN diagram illustrating the first workflow process for utilising ToleranceDM. Organisations develop, maintain, and iteratively refine DSM and feature maps to be used in the second workflow process.***

The second process (Figure 3) is undertaken by BIM engineers on a project-specific level. The two output artefacts from the first workflow process, namely the DSM and feature map, are combined with the BIM of a particular project at hand in order to generate the project-specific set of tolerance sensitive relations between products. This is accomplished automatically by the ToleranceDM Reasoning Engine; in this paper we have developed a prototype system as a proof of concept, and to demonstrate analysis functionality provided by our ToleranceDM approach.

The newly created tolerance sensitive information is specifically expressed in the form of new BIM relationships injected into the model: new instances of the class "Tolerance Sensitive Relation" that hold between various products in the BIM. The particular subclass relationship determines the refined semantics of the tolerance relationship, as detailed in Figure 4, Figure 6 and Table 1. Based on these new relationships, various metrics are computed that provide building information modelling engineers and project managers with new information about the planned construction from the perspective of tolerance sensitivity. The analysis is tailored by the building information modelling engineers in the form of "tolerance queries" (or "filters"), which we exemplify in the case study in Section 5.

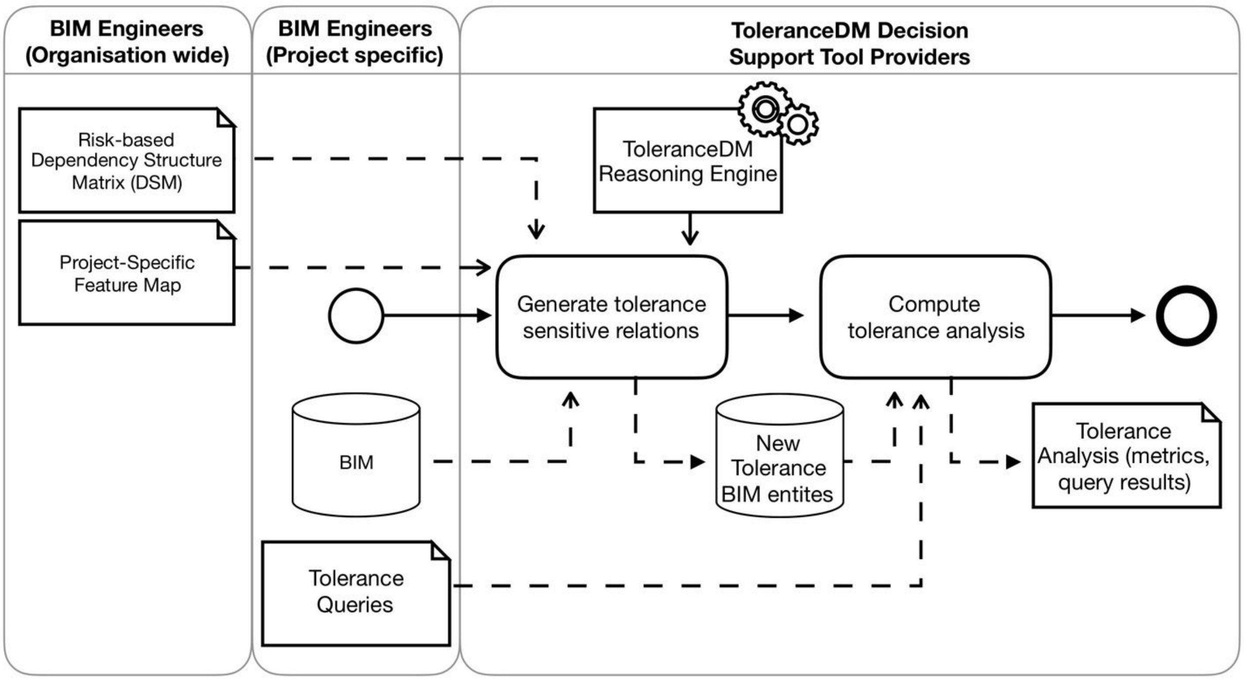


***Figure 3: BPMN diagram illustrating the second workflow process for utilising ToleranceDM with respect to a specific project.***

In this study we propose a rule-based reasoning approach and a prototype software system for deriving, querying, and analysing tolerance relationships in large-scale BIM models. We demonstrate the practical aspects of our implementation on a real use case and emphasize modular and configurable rule encodings so they can be easily adapted to *general* tolerance management scenarios.

### 4.1 An Abstract Structure of *Tolerance Management*

In this section, definitions are provided for the most general concepts and their relations with respect to tolerance management (from an ontological perspective), and a selection of class refinements derived from literature and other resources described in Section 3. This provides a conceptual language for semantically enriching a BIM, and subsequently facilitating tolerance management querying and analysis.

As illustrated in Figure 4, the most abstract structure of tolerance management concepts consists of *Tolerance Sensitive Relations* that hold between BIM entities via spatial relations. If such a relation holds between entities, then deviations during manufacturing, placement, and aggregation of those entities beyond a threshold will have a specific, negative impact on the overall project. In this research, the term *BIM entity* is further discretized into components (i.e., construction objects that aggregate into assemblies), and component-parts, herein referred to as parts (i.e., specific geometric aspects of a component such as the top surface of a beam). The definition of which entities are in a tolerance sensitive relation is the formalisation of knowledge from the tolerance community. For example, as shown in Figure 5:

- the notion that "*deviations in the placement of two adjacent slabs can result in a dangerously large gap*" is expressed by assigning a (binary) tolerance sensitive relation between slabs that meet flush horizontally (i.e., vertical faces of the defining geometries "touch", or have external contact).
- the notion that "*deviations in the parallel alignment of two beams that frame an opening for a door can result in the door no longer fitting*" is expressed by assigning tolerance sensitive relations

between any two beam entities (the entity types) that both have spatial contact (the spatial relation) with the same door entity.

- the notion that "*deviations in the flatness of a slab can result in unwarranted slopes for mobility impaired patients in a hospital*" is expressed by assigning (unary) tolerance sensitive relations to slabs in locations occupied by such patients.

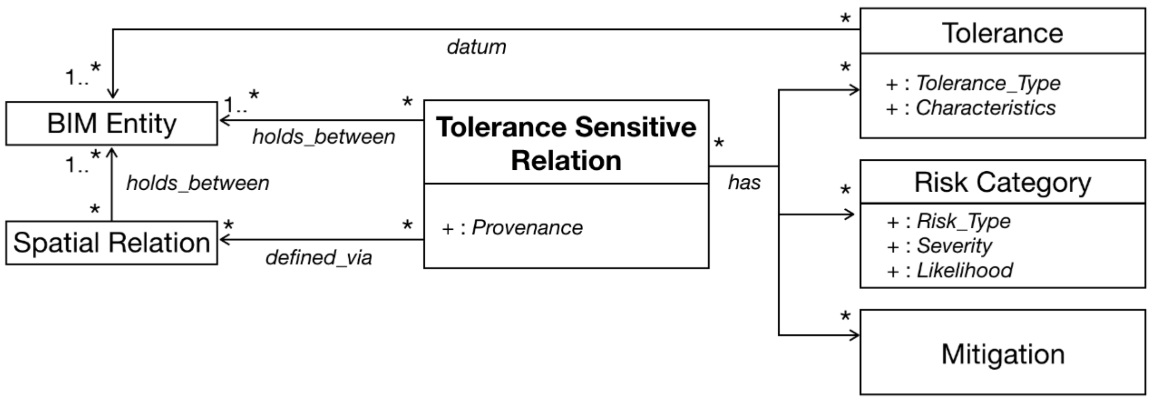


***Figure 4: Abstract structure of high-level tolerance management concepts.***

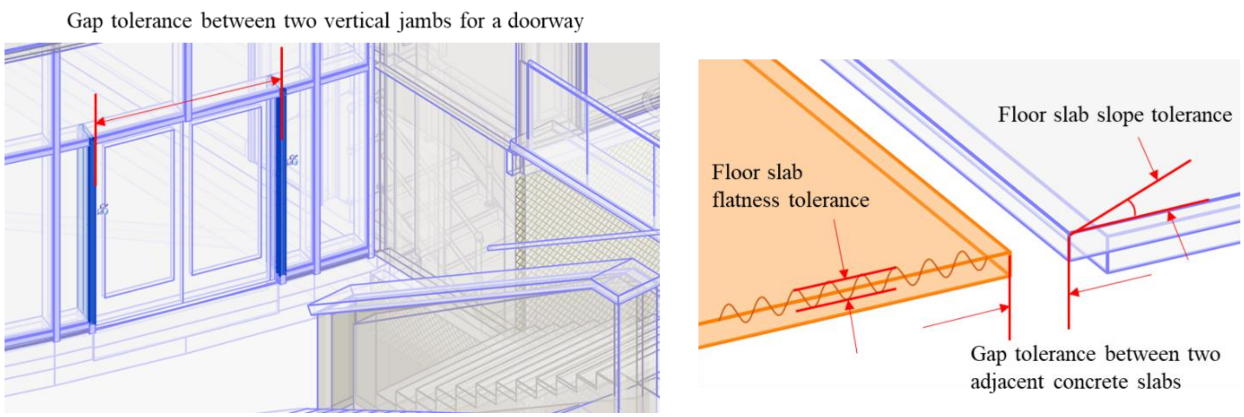


***Figure 5: Demonstrating how tolerance sensitive entities can be formalised.***

These tolerance relations are introduced into a BIM on the same ontological level as other entity relations such as *contact* between two entities that have physical contact, or *voids object* between a wall and an opening i.e., a region of empty space. The *arity* of a tolerance sensitive relation specifies the number of entities between which the relation holds. A unary relation holds over a single entity, equivalent to the concept of an entity property, e.g., the tolerance sensitive quality of slab flatness is modelled as a unary tolerance sensitive relation holding over individual slabs.

Each tolerance sensitive relation has attributes of *provenance* and *risk*:

- *Provenance* refers to the source of the tolerance sensitive relation class: *normal* (derived from guidelines and standards); *special* (based on expert input for certain parts); *specific* (based on expert input for the entire building structure) [47].
- *Risk* refers to the frequency and impact of tolerance conflicts.

Each tolerance sensitive relation is associated with:

- *Tolerance* constraints that define the limits within which deviations will not result in risk category impacts, as defined in Section 2.1 and 2.2. Table 1 presents further details on the information captured in the Tolerance class, adapted from the GD&T ontology [48]. In Table 1, the concept of datum as part of GD&T refers to a theoretically exact axis, point or plane from which the geometric characteristics or location of a feature (e.g., size or surface) are established [29].
- *Risk Category* effects that result from tolerance conflicts, as defined in Section 2.2, consisting of a risk *type* (structural safety, constructability, aesthetic, functionality) qualitative measures of severity and likelihood (low, medium, high, extreme).
- Mitigation strategies that can be employed to alleviate the impact of risk categories in case of tolerance conflicts.

***Table 1: Information expressed in the ToleranceDM Tolerance Sensitive Relation class, adapted from GD&T ontology [48].***

| Tolerance Type | Characteristics | Datum Req'd? | Applications |
|---|---|---|---|
| **Size:** spatial extent of an entity | Dimensions | No | To control dimensions of any component |
| **Form:** the shape of a surface. | Straightness: allowed deviation of a surface along a line. | No | To control the beams and columns that are prone to deformation. |
| | Flatness | No | To control the flatness of floor surfaces. |
| **Orientation:** relationship between features and datums at particular angles. | Perpendicularity (surface): variation allowed over a form being parallel to the datum plane. | Yes | To control components for which plumbness tolerances are a major concern. |
| | Parallelism: variation allowed over an entire plane, from being parallel to the reference plane. | Yes | To control surfaces that should maintain a constant distance. |
| **Location:** position of the feature relative to a datum. | Tolerance of Position (TOP): allowed deviation of a feature's axis from the True position. | Yes | To control (1) the location of features of size such as columns and beams, and (2) the distance between those features of size |

In the following sections, we define particular refinements (subclasses) of the Tolerance Sensitive Relation class.

## 4.2 Tolerance Interdependency Matrix

To capture the interdependency of components and parts with respect to tolerances, a dependency structure matrix is used. This matrix is referred to as the *Tolerance Interdependency Matrix* since it outlines the direct relationships within and between components and parts. The tool is versatile with respect to the LOD that components are related, e.g., it is used to express dependencies between component types such as foundations and walls, and dependencies between component parts such as the top surface of foundations in contact with the lower surfaces of a wall.

Assembly components and parts are listed both in the rows and columns of the matrix. Each component and part are represented with the symbolic notation $C_T^P$ with subscript $T$ indicating the component type and superscript $P$ indicating the part. If there is any physical connection (i.e., joint or interface) between the component (and part) at row *i*, and component (and part) at column *j*, the interrelated cell is filled with a value according to its tolerance type; otherwise, it is filled with 0. The types of tolerances considered here include flatness (F), parallelism (PA), positional (PO), and perpendicularity (PE). The process of capturing tolerance interdependency can be formalized algebraically as follows. First, let $M$ denote an $n \times n$ Tolerance Interdependency Matrix, and let $m_{ij}$ denote the tolerance interdependency at the $i$-th row and $j$-th column, for $i, j \in \mathbb{N}$, such that $1 \leq i,j \leq n$. For all $i, j$ from 1 to $n$:

- $m_{ij} \subseteq \{F, PA, PO, PE\}$, if $C_i$ has a tolerance interdependence with $C_j$.
- $m_{ij} = 0$, otherwise

The rows and columns are ordered according to the temporal sequencing in which components are installed with respect to preceding components. The matrix, when populated with binary relations, is not necessarily symmetric because the sequence of installation/erection of components is also being captured. Furthermore, tolerance interdependencies which are self-intersecting can be used to denote that a component or part has an intra-object tolerance relation (e.g., flatness).

Once the matrix is populated and the interdependencies are captured, a final additional step is to colour the cells based on predefined levels of risk which can be defined by (1) the collective decision of project representatives responsible for tolerance management, and/or by (2) predetermined risk of connections found from the literature [13]. The basis of this risk colorization is representative of adversarial impacts on structural safety, constructability, aesthetics, and functionality. For instance, red denotes high tolerance risk, orange denotes medium tolerance risk, and green denotes low tolerance risk. Based on this categorization, it is possible to capture risk associated with connections as follows:

- **Low risk connections**: Remedial costs are non-negligible (yet not significant), and the functionality is adversely affected but the functionality of the connection still conforms to the specifications.
- **Medium risk connection**: Remedial costs are higher, and the functionality is adversely affected in such a way that the connection does not function as intended.
- **High risk connection**: Remedial costs are highest and there is a high risk to safety.

Table 2 depicts the notation for developing the Tolerance Interdependency Matrix. The left column denotes preceding component sequencing while the top row denotes succeeding component sequencing. For a sample completed Tolerance Interdependency Matrix, the reader is directed to Appendix A which corresponds to the case study presented in this paper.

***Table 2: Tolerance Interdependency Matrix Notation***

| | | $C_1$ | | | $C_2$ | | | $C_3$ | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | $C_1^1$ | $C_1^2$ | $C_1^n$ | $C_2^1$ | $C_2^2$ | $C_2^n$ | $C_3^1$ | $C_3^2$ | $C_3^n$ |
| $C_1$ | $C_1^1$ | $m_{11}$ | … | … | … | … | … | … | … | $m_{1j}$ |
| | $C_1^2$ | … | … | | | | | | | |
| | $C_1^n$ | … | | … | | | | | | |
| $C_2$ | $C_2^1$ | … | | | … | | | | | |
| | $C_2^2$ | … | | | | … | | | | |
| | $C_2^n$ | … | | | | | .. | | | |
| $C_3$ | $C_3^1$ | … | | | | | | … | | |
| | $C_3^2$ | … | | | | | | | … | |
| | $C_3^n$ | $m_{i1}$ | | | | | | | | $m_{ij}$ |

A fully developed serialization of a DSM is future work. For the initial version, we serialise DSMs in a comma separate format (CSV). Column and row headers are represented using the format (with all spaces in labels replaced by underscore "_"):

```
<Component> : <Part>
e.g., "Concrete_Slab : Top_Surface"
```

Cell entries are a list of symbols in square brackets separated by semicolons based on a simple coding system: F= Flatness tolerance, PA: Parallelism tolerance, PO: Position tolerance, PE: Perpendicularity tolerance (e.g., [F,PO] means the corresponding cell entry has flatness and position tolerance sensitivity"). For example, the first three columns and rows of the example DSM in Appendix A is serialised in CSV format as follows:

```
Foundation : Top_Surface, Concrete_Slab : Top_Surface, Concrete_Slab : Bottom_Surface
[], [], [F]
[], [], []
[F], [], []
```

## 4.3 Operationalizing ToleranceDM

In this section, we describe the task of s*emantic enrichment* for augmenting a BIM with tolerance sensitive relations. These new relations are then used to provide project stakeholders with a range of *query services* concerned with identifying components in the BIM for which tolerance is a critical concern.

### 4.3.1 BIM Semantic Enrichment

Within the context of BIM, a building component (i.e., an "object") consists of a type, e.g., IfcDoor, and a unique identifier, e.g., a GUID. Components can be assigned to one or more geometric representations, e.g., IfcRepresentation including 2D/3D surface representations etc. An *n*-ary relation has a type, e.g., IfcRelVoidsElement, and is assigned to *n* components, meaning that the relation holds between those components.

An instance of a BIM, denoted $B$, consists of a set of building components $C$ and relations $R$ between those components, $B = C \cup R$. Each component in $C$ belongs to a hierarchy of building entity class types, e.g., IFC classes IfcRoot, IfcProject, IfcDoor etc. Each component has zero or more geometric representations such as 3D meshes, 2D polygonal footprints, etc. This research denotes a geometric representation $g$ of component $c$ using the predicate rep($g$, $e$). Let $S_1,...,S_m$ be a set of predefined spatial relations defined over specific types of geometric representations such as intersects, meets_flush, is_flat, etc. For brevity, a tuple of building components $c_1,...,c_k$ in $C^k$ (for $k > 0$) is considered to satisfy a given spatial relation $S_i$, denoted $S_i\,(c_1,...,c_k)$, if their corresponding geometric representations satisfy the spatial relation, $S_i\,(g_1,...,g_k)$ such that rep($g_j$,$e_j$) for $1 \leq j \leq k$.

*BIM augmentation* consists of identifying new tolerance sensitive relations that hold between tuples of components in $C$, and adding these relations to the set of BIM relations $R$. In this first version of ToleranceDM, a general "ToleranceSensitiveRelation" relation is defined within the IFC standard. As shown in Figure 6, the tolerance sensitive relation is a subclass of IfcRelConnects; in keeping with IFC naming conventions the new relation is called *TmsRelToleranceSensitiveConnects*.

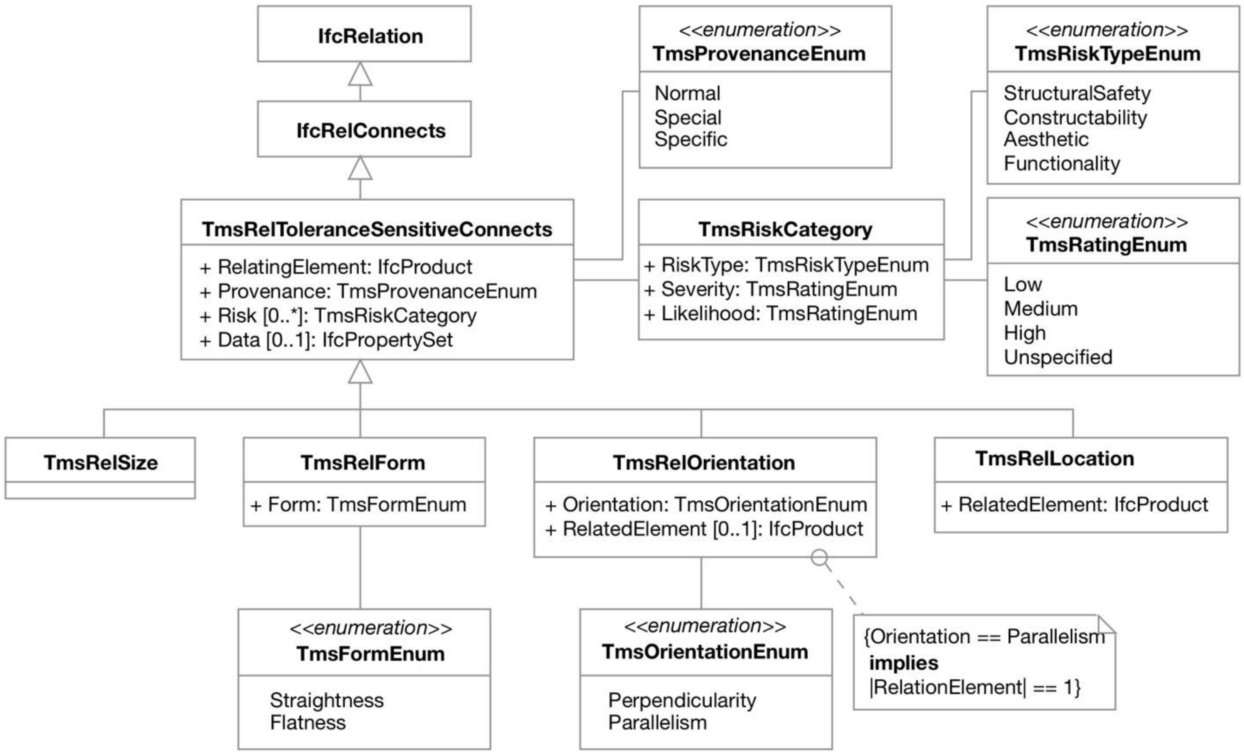


***Figure 6: UML class diagram depicting that the new TmsRelToleranceSensitiveConnects relation(and the refining subclasses are a subclass of IfcRelConnects within the IFC standard.***

The IfcRelConnects relation has been chosen as a suitable superclass owing to the semantics of that IFC class expressing "a connectivity relationship that connects objects under some criteria [such that] subtypes of the relationship define the applicable object types for the connectivity relationship and the semantics of the particular connectivity" [49].

IfcRelConnectsElements was also a viable candidate, however this was rejected as it is constrained to be a *binary* relationship that always holds between exactly two entities. Tolerance sensitivity, as an abstract relationship, requires more flexibility. For example, "flatness" is more appropriately modelled as a *unary* relation.

IfcPropertySets were also under consideration as a means of integrating ToleranceDM into IFC. However, property sets assign properties to products, rather than expressing new relationships *between* products. ToleranceDM takes the modelling stance that "tolerance" as a concept is captured most appropriately, with respect to semantics as a relationship that can hold between products. That is, many kinds of tolerances are relative between product instances, e.g., "parallelism" is a tolerance on the relative orientation of two products.

In terms of scope for the first version of ToleranceDM, the assignment of numerical values to tolerance parameters is kept at a general level (also referred to as tolerance *allocation* in product design), e.g., specifying an exact numerical positional distance threshold for a particular TmsRelLocation relation instance. The issue from a modelling perspective is that various construction projects need significant flexibility in how numerical information is to be interpreted. For example, considering "flatness" one option would be to enforce a "flatness" metric to define a threshold, although any chosen metric may conflict with an organisation's preferred standard approach. Thus, rather than enforcing metrics and structure prematurely, all TmsRelToleranceSensitiveRelations have an optional "Data" attribute that refers to an IfcPropertySet. This provides a high level of versatility and may later be revised when further evidence has been collected on the most useful ways of introducing more structure into these metrics, from industry practitioner interaction and case study analyses (as part of the IDEF5 refinement and validation modelling development stage).

As described in Section 3.4, a more detailed treatment of mitigation is left for future version of ToleranceDM and is thus mitigation concepts are not integrated into IFC in this first version. Also, TmsRiskCategory is a subclass of IfcRoot in this version of ToleranceDM. This relationship was omitted from the diagram to reduce visual clutter.

The first version of ToleranceDM supports 3D meshes as the geometry representations for components. A uniform treatment of disparate geometric representations from a BIM/IFC is achieved through IfcOpenShell's default Delaunay triangulation. The geometry processing typically takes a few minutes for a BIM with $10^4$ objects but only must occur once. A 3D point $p_i$=($x_i$,$y_i$,$z_i$) is a 3-tuple of real numbers. A 3D triangle is three vertices defined by distinct 3D points $p_1$, $p_2$, $p_3$. A 3D mesh $g$ is a set of 3D triangles referred to as *faces*. The distance between two meshes $g_1$, $g_2$ is the minimum distance between every pair of triangles $t_i$,$t_j$ such that $t_i$ is a face in $g_1$, and $t_j$ is a face in $g_2$.

### 4.3.2 Inferring Contact in BIM

In this version of ToleranceDM, it is assumed that semantic contact information has been omitted from the original IFC, e.g., no products are assigned the IfcRelConnectsElement relation, due to a modelling oversight. If such semantic information is available then the task of introducing tolerance relations is simplified, as semantic contact does not need to firstly be inferred based solely on the geometry of the products.

A key challenge in tolerance-based model augmentation is inferring spatial contact between components. The primary source of information used to infer contact is the geometry associated with BIM components. However, contact between two components *does not imply* that their geometric representations have zero distance between them in a BIM:

- In reality, a building is not static, and components continuously deform (e.g., expand or shrink). The BIM designer may therefore intentionally leave a gap between two components with contact so that any building movement would not lead to physical clash between them.
- The BIM may be modelled at a reduced LOD meaning that certain joining components (e.g., bolts, gusset plates, etc.) are omitted. For example, Figure 7 illustrates a BIM (referred to as "Simple Modular") in which brace components are intended to connect to beams in the design. However, owing to the selected LOD by the BIM designer, the geometric representations are disconnected (i.e., having a minimum distance greater than zero).
- The exact placement of components may not be precisely known, especially in the earlier stages of design. In this case, the geometry in a BIM may only be a coarse approximation of the intended final placement, and thus, geometries of two components may be disconnected although the designer intends that they have contact. This is also reflected in different model view definitions (MVDs) that may be used in a project – given certain views, some components will be omitted from an MVD, resulting in contact being more complicated to infer explicitly from the raw geometry during model exchange between stakeholders.

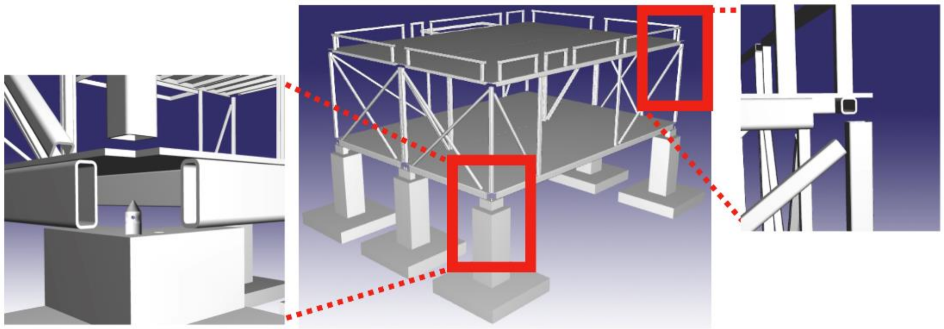

***Figure 7: Example "Simple Modular" BIM in which components that are intended to have contact have geometric representations that are disconnected. Interpreted literally, the brace components are not resting or attached to any other component, and would fall, making the BIM non-constructable.***

An automated tolerance management software tool that has industry value and impact must nonetheless be capable of inferring the designer's *intention* of spatial contact between such components in the design. Thus, the inference of a contact relation between two components is only a *hypothesis,* justified by *evidence* in the form of the given geometric representations in the BIM, and background knowledge about physics, building construction, and BIM levels of development. For clarity, we distinguish these concepts with the terms *intended contact* (i.e., cases where the designer intends components to have contact) and *geometric contact* (i.e., when the 3D geometric representations of two components has zero distance between them)[1].

Therefore, despite BIM components not strictly achieving geometric contact, it is argued that the intention of contact is represented by information in the given (disconnected) geometric configuration and the semantics of the component types, within the context of tacit knowledge about buildings. For example, a knowledgeable construction engineer will be confident that certain braces illustrated in Figure 7 are intended to have contact with certain beams due to:

- the spatial arrangement of the components (proximity, orientation).
- the component types (i.e., braces typically connect with beams).
- the necessity of their contact for a coherent BIM - i.e., if they did not have contact, then the parts would be illogically "floating" in free space and thus the BIM would not be stable, safe nor constructible.

The task of hypothetically inferring contact relations can be addressed within a framework of logical abduction. For simplicity, in this first version, an optimistic (or aggressive) inference policy is adopted. That is, if the distance between two component (or part) geometries is within a specified threshold then contact is hypothesized. Based on the LOD framework used in this research [19], a minimum level of development required for ToleranceDM is LOD 200. At this level, elements are considered to be generic geometric placeholders, but may be recognizable as the components they represent particularly for rectilinear objects such as concrete footings or steel HSS members. At LOD 100, elements are not considered to be geometric representations and therefore there is not enough information to infer contact between components.

[1] These concepts are different from the semantic relation IfcRelConnects which does not imply geometric or intended contact. IfcRelToleranceSensitiveConnects is derived from a BIM using abduction, e.g., if they are justified by background knowledge and consistent with current observations. Our connection finding algorithm is designed to be modular and configurable, so users can refine it by object type, path criticality, node connectivity, etc. and can optionally introduce all "found contact" relations as new instances of IfcRelConnects.

Having generated contact relations, a contact graph is subsequently derived by taking components (or parts) as nodes, and contact relations between components (or parts) as edges between the corresponding nodes. A set of contact relations is *valid* if the contact graph is connected, i.e., there is a path through the contact graph between any pair of nodes. If the set of contact relations is *invalid*, then some components may be "free-floating" without ultimately being supported by foundation components, suggesting that either necessary contact relations have erroneously not been inferred, or that the BIM cannot be constructed.

For example, Figure 8 illustrates a derived contact graph visualised together with the geometry of the components. Components are represented by small red boxes (graph nodes) and contact between components is represented by thin blue cylinders (graph edges). Nodes have been placed approximately in the centre of the corresponding component mesh (calculated as the mean of the mesh vertices). In the right subfigure, red nodes can be seen to not have any connecting edges and thus this particular set of hypothesized contact relations is invalid, requiring the distance threshold for inferring contact to be increased.

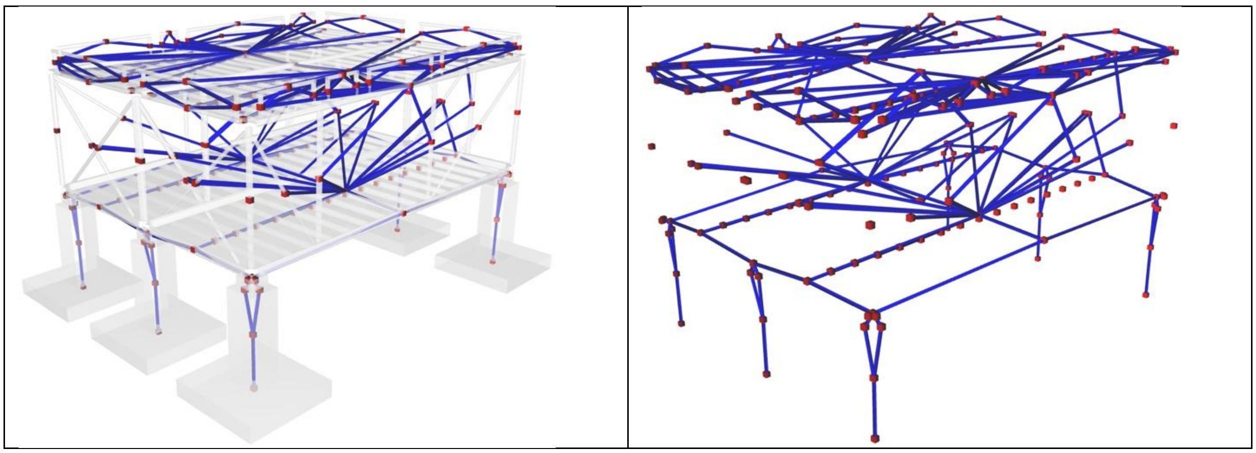

***Figure 8: Derived contact graph visualised with the "Simple Modular" BIM geometry (left) and the graph visualised in isolation (right).***

#### 4.3.3 BIM Tolerance Analysis and Querying

Having inferred contact relations, a Tolerance Interdependency Matrix is used to augment a BIM with tolerance sensitive relations. The matrix identifies connections and tolerance risks in those connections. The Tolerance Interdependency Matrix utilised to develop ToleranceDM can be re-used as a benchmark for next projects of the same type. Users can then add/remove connections or change the associated tolerance risks based on the type of the project, thus saving time required for the initial setup of ToleranceDM. The pseudocode algorithm is as follows.

***Algorithm 1. Augmenting BIM with Tolerance Sensitive relations.***

```
Input: Matrix M, Building_Information_Model BIM
Output: Building_Information_Model

For each cell in M with value other than "0":
  Get the component/part class type label of the cell's row (C1)
  Get the component/part class type label of the cell's column (C2)
  For each instance c1, c2 in BIM component/parts C1, C2:
    If c1, c2 have contact then:
```

| |
|---|
| **6: Generate a new tolerance sensitive relation between instances $c_1$, $c_2$ (with risk and impact information expressed in the Matrix) and add to BIM**<br>**7: return BIM** |

For example, Table 3 presents a Tolerance Interdependency Matrix that consists of just one cell indicating that the contact between slabs and columns has a high risk (i.e., red colour) – in this case corresponding to a positional tolerance (i.e., "PO"). Information about the specific impact is also optionally encoded in the Matrix, i.e., the example Matrix indicates that the risk is to constructability. Each instance $c_1$, $c_2$ of an IfcSlab and IfcColumn such that they have contact (as derived in the previous section) results in a new tolerance sensitive relation being created between $c_1$, $c_2$.

Figure 9 illustrates an extract of the knowledge graph that shows how the original IFC model is augmented with the new tolerance information in the form of relations and risk category entities. Figure 10 illustrates the application of the DSM in Table 3 to the modular BIM by colouring all components red that occur in a high-risk tolerance sensitive relation.

***Table 3: An example Tolerance Interdependency Matrix.***

| | IfcColumn |
|---|---|
| IfcSlab | PO (constructability) |

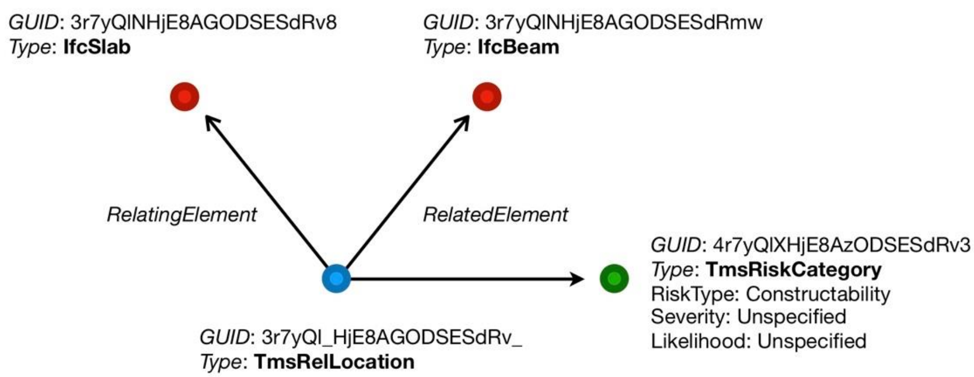


***Figure 9: Extract of knowledge graph to illustrate how the BIM is augmented with tolerance information. The original IFC file contained two products represented by the red nodes in the graph (IfcSlab and IfcBeam). The ToleranceDM reasoning system determined these products to have contact and based on the DSM (Table 3) the new tolerance sensitive relation TmsRelLocation is injected into the BIM (blue node) with its associated RiskCategory entity (green node).***

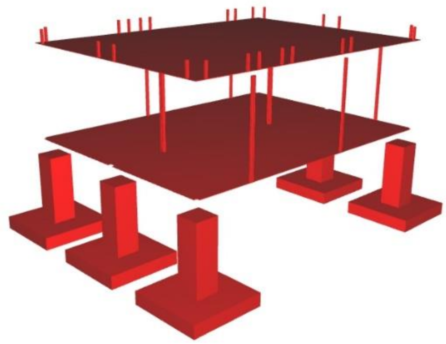

***Figure 10: Visualising components in the "Simple Modular" BIM that occur in a high-risk tolerance sensitive relation in red, according to the Matrix presented in Table 3.***

Given an augmented BIM, a wide range of queries Q1-Q6 can be executed:

- Q1: Finding all components/parts that occur in an intended contact relation.
- Q2: Finding all components/parts that occur in a tolerance sensitive relation.
- Q3: Filtering Q1, Q2 according to risk level (low, medium, high).
- Q4: Filtering Q1, Q2 according to risk type (i.e., structural safety, constructability, aesthetic, functionality).
- Q5: Filtering Q1, Q2 according to component/part types (e.g., slabs, columns, etc.).
- Q6: Applying any combination of filtering options Q3-Q5.

Statistics can also be derived based on the above query results, as enumerated in S1-S3 below. For example, for a given component, one can count the number of other components that it has contact with, referred to as the degree of contact (i.e., using graph theory terminology in the context of the contact graph).

- S1: number of contact relations that a component/part occurs in (degree of contact)
- S2: number of tolerance sensitive relations that a component/part occurs in (degree of tolerance sensitivity)
- S3: taking the results from Q6 and applying S1, S2 (degree of query-refined contact or query-refined tolerance sensitivity)

Figure 11 illustrates a scatterplot comparing contact degree with component type for the "Simple Modular" BIM (corresponding to statistic S1). Each data point ("x" symbol) is a component in the BIM. The X-axis categorises components according to their type (e.g., slabs, columns, etc.). The Y-axis measures the degree of contact, i.e., the number of other components that a given component (marked "x") has contact with.

As an example of the analytical support that ToleranceDM provides, the scatterplot shows that slabs (as a class type) have both the highest contact degree, and the largest range of contact degrees, followed by beams, columns, members, and building element proxies. This suggests that slabs make up the contact "hubs" of the construction. Buildings, building storeys and the site have no explicit geometric representation in the given BIM, and thus have no contact relations[2].

Figure 11 also illustrates the relative degree of contact between components (corresponding to statistic S1). Components visualised in red have a relatively high degree of contact (e.g., the central slabs), and components visualised in blue have a relatively low degree of contact (e.g., foundation slabs and lower columns).

To contrast contact and tolerance sensitivity, Figure 12 illustrates a scatterplot comparing tolerance sensitive degree and component type in the "Simple Modular" BIM, based on the Matrix in Table 3, corresponding to statistic S2. Comparing Figure 11 and Figure 12, it is observed that Figure 12 illustrates the subset of contact relations that correspond to an entry in the DSM in Table 3. The colour gradients have been recomputed to visually reflect the new range of degrees, i.e., from low (blue) to high (red) *tolerance sensitivity* based on degree, as compared to *contact*.

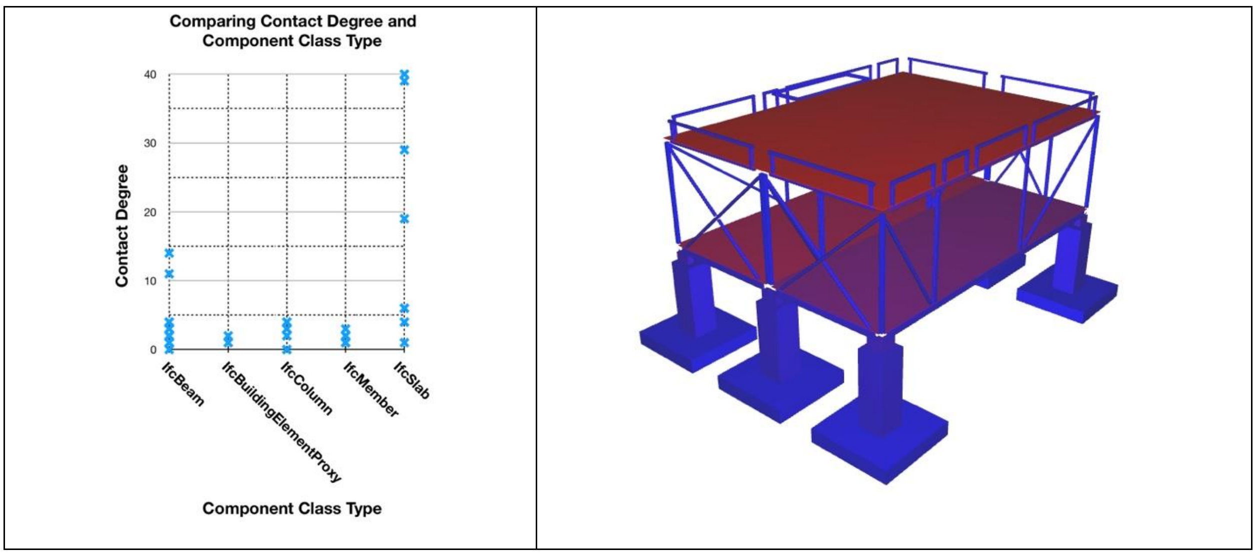


***Figure 11: (left) Scatterplot comparing contact degree with component class type for the "Simple Modular" BIM. (right) Using a colour gradient (from red to blue) to visualise the relative degree of contact of each component, where red corresponds to a high degree.***

[2] Components are organised into building storeys through the IfcRelContainedInSpatialStructure relation (and, in turn, storeys are composed to form the whole building through IfcRelDecomposes relation). The building storey could be assigned to the set of all geometric representations of contained components, providing some interpretation of contact between building storeys, buildings, and other components.

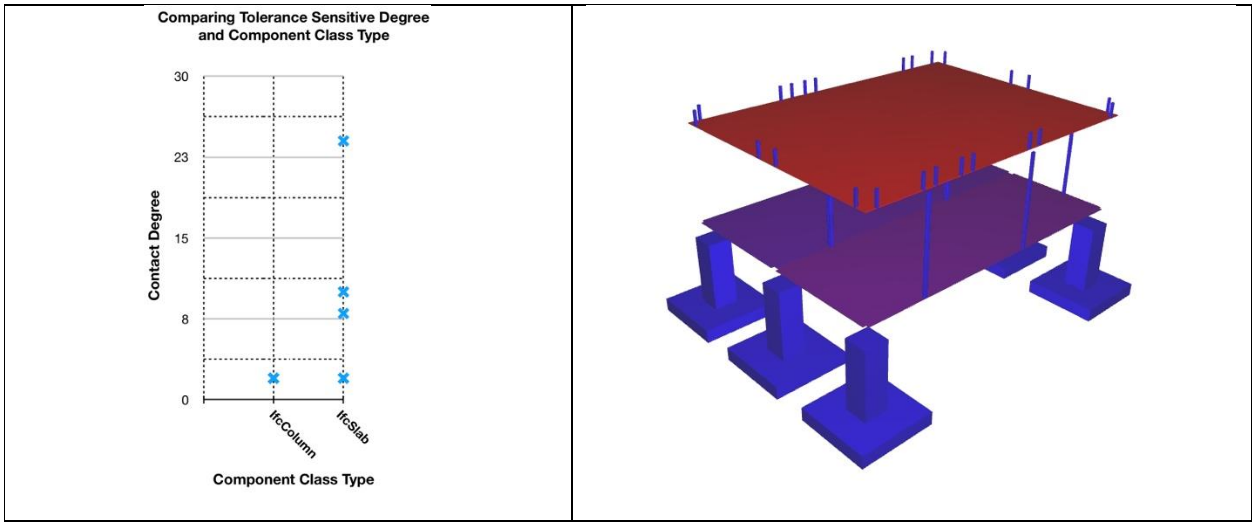


***Figure 12: (left) Scatterplot comparing tolerance sensitivity degree (based on the Matrix in Table 3) with component class type for the "Simple Modular" BIM. (right) Using a colour gradient (from red to blue) to visualise the relative degree of tolerance sensitivity of each component, where red corresponds to a high degree.***

## 5.0 Functional Demonstration

This functional demonstration of ToleranceDM focuses on consolidating tolerance domain knowledge pertaining to BIM LOD 300 (since there are instances where both BIMs do not contain parts necessary for coordination of the element with nearby of attached objects, they cannot be considered to be fully within the LOD 350 designation). Two BIMs are used in this demonstration. The "Simple Modular" BIM (presented in Section 4.3) was developed for a prototypical modular construction project in Canada. Due to the simplicity of its building assembly, this BIM is investigated in this paper as a working model for developing ToleranceDM. The second BIM ("Oastler") is a large public building with steel structure, composite steel deck-slabs, curtain walls, glazing, masonry panels, suspended floors and a roof. All ground floor slabs are comprised of reinforced concrete and bear directly onto the ground. The cost for this 7,500 $m^2$ building was approximately $37.4M. The bespoke curvature of the building and anodized aluminum fins attached to the structure, give it a distinctive architectural feature, while adding important tolerance sensitive connections. Table 4 presents geometry and model statistics of the two BIMs – of particular note is the difference in BIM sizes, both in the number of building components, and the complexity of geometric representations as 3D meshes, thus reflecting the ability to gauge the scalability of ToleranceDM.

***Table 4: Geometry and contact relation statistics for the demonstration BIMs.***

| BIM | Components | Meshes | Triangles | Inferred contact relations | Contact degree | | |
|---|---|---|---|---|---|---|---|
| | | | | | Min | Mean | Max |
| Simple Modular | 211 | 206 | 54,864 | 252 | 0 | 2.45 | 40 |
| Oastler | 3,355 | 3,330 | 678,860 | 13,514 | 0 | 8.12 | 515 |

IFC BIMs were parsed using the IfcOpenShell tool suite,[3] to extract component unique identifiers, component class types, and 3D meshes. The reasoning engine was developed using a combination of the logic programming languages Answer Set Programming and Prolog, based on the InSpace3D system [50]. ASP and Prolog constitute the theoretical foundation of our software system in that every inference can be logically derived, thus *verifiable* and *explainable*. This is particularly important in the scope of tolerance management due to irreducible deviations from BIM geometry (two components are almost touching) from design intents (the components should be connected). Therefore, each tolerance relationship is contingent to a number of default assumptions and educated guesses about tolerance relationships. ASP and Prolog provides modifiable and extensible encodings for this additional knowledge in the form of defeasible rules, so the process of augmenting BIM with tolerance relationships is transparent and verifiable.

Firstly, the BIM was parsed into logic programming facts of the following form representing each component unique identifier and class type:

```
product(id("2aBHA8A3r75w4VRqHJ3d9h"),type(ifcBuilding)).
product(id("3r7yQlNHjE8AGODSESdRmc"),type(ifcBeam)).
...
```

The BIM geometries were parsed into facts about geometric representations of 3D meshes, consisting of 3D point vertices, triangular faces, and face-normals (which are used to determine whether a given face is part of the top, bottom or sides of a product geometry):

```
representation(surface_mesh,
            id("3r7yQlNHjE8AGODSESdRmc"),
            mesh(vertices([point(-6.964,-14.705,0.488),...]),
                faces([face(11,63,10),...]),
                face_normals([ vector(0,1,0),...])
            )).
```

The 3D axis aligned bounding box of each mesh was derived, represented as two 3D points (the left lower front corner, and right upper back corner). These are expressed as facts such as:

```
representation(axis_aligned_box3d,
            id("3r7yQlNHjE8AGODSESdRmc"),
            box3d(point(-6.964,-17.852,0.304),
                 point(-6.888,-14.705,0.507))).
```

Bounding boxes were scaled and translated into positive integer representations for more efficient processing:

```
aabb("3r7yQlNHjE8AGODSESdRmc",(8001,10189,24891,8657,37295,26641)).
```

Contact was hypothesized based on bounding box intersection. Approximately 13,000 contact relations were derived out of approximately $5.4 \times 10^6$ distinct candidate pairs (i.e., where every pair of meshes is a candidate hypothesis, resulting in $N(N-1)/2 \approx 5.4 \times 10^6$ with $N$=3300). Two boxes intersect if their projections intersect in all three axes, as implemented in the following ASP program:

```
%% Points of bounding box projected onto each axis
projection(Id, x, X1; Id, x, X2; Id, y, Y1; Id, y, Y2; Id, z, Z1; Id, z, Z2) :-
   aabb(Id, (X1, Y1, Z1, X2, Y2, Z2)).

%% Holds if either start/end point of Id1, projected onto Axis, is within
```

[3] http://www.ifcopenshell.org/

```
%% the interval defined by Id2 points projected onto Axis.
point_intersects_line(Id1, Id2, Axis) :-
   projection(Id1, Axis, M),
   projection(Id2, Axis, Min), projection(Id2, Axis, Max),
   Min <= M, M <= Max.

%% As above, but with Id1 and Id2 swapped
point_intersects_line(Id2, Id1, A) :-
   projection(Id1, A, M), projection(Id2, A, Min), projection(Id2, A, Max),
   Min <= M, M <= Max.

%% Components Id1,Id2 have contact if their bounding boxes intersect
%% in all three axes (x,y,z)
contact(Id1, Id2) :-
   aabb(Id1, _), aabb(Id2, _),
   Id1 < Id2,  %% standard ASP approach to avoid symmetric and reflexive cases
   point_intersects_line(Id1, Id2, x),
   point_intersects_line(Id1, Id2, y),
   point_intersects_line(Id1, Id2, z).
```

Figure 13 illustrates the derived contact graph for the Oastler BIM.

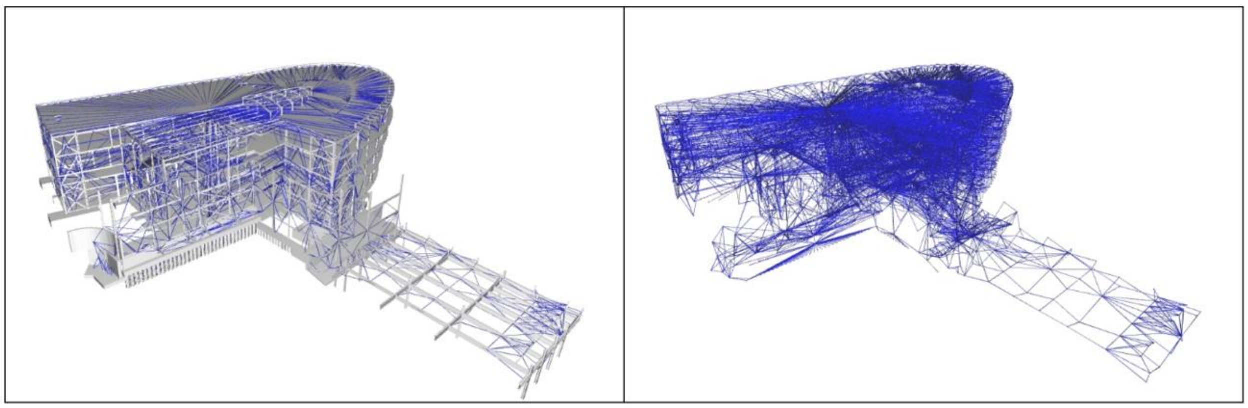

***Figure 13: Derived contact graph visualised with the "Oastler" BIM geometry (left) and the graph visualised in isolation (right).***

A tolerance interdependency matrix was created for the Oastler functional demonstration (Appendix A). There are three main steps used to infer the tolerance sensitive relations specified in the matrix. As described previously, the first step is to infer spatial contact relations between all components (discussed in more detail in Section 6.1). Class types such as foundations, stone cladding, internal partitions, and office doors were not explicitly defined in the given IFC of the Oastler building, and thus for the second step we hand-crafted a simple set of rules tailored to the Oastler BIM to identify which objects belonged to the relevant project-specific class. The third step consisted of deriving tolerance sensitive relations according to the class types specified in the matrix, and spatial contact relations inferred in the first step.

Figure 14 illustrates the fragments of the BIM that correspond to each cell in the matrix that is assigned a tolerance sensitive type. In total, 10387 tolerance sensitive relations were inferred: 7063 flatness; 2173 position; 189 perpendicular; 962 parallel.

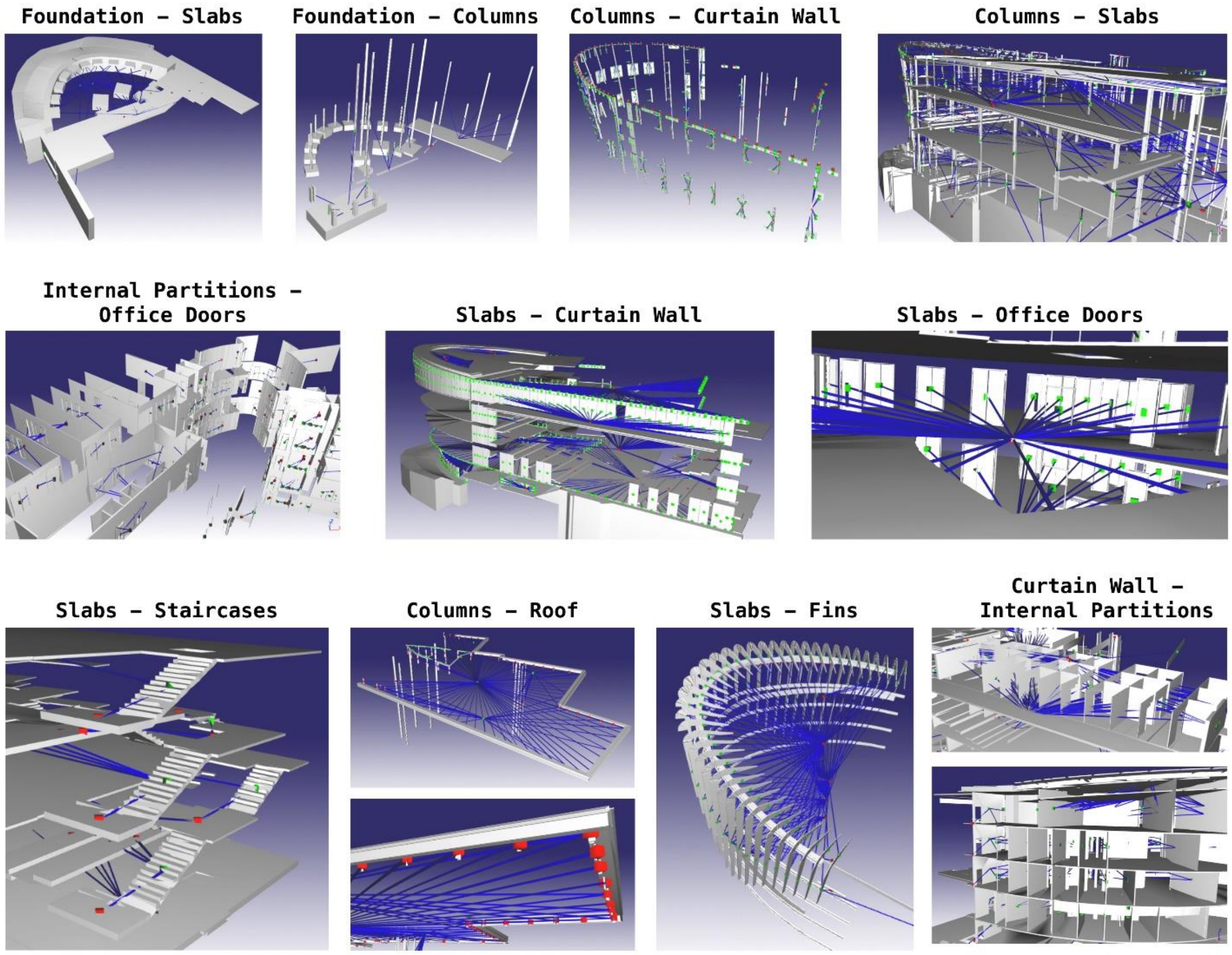


***Figure 14: Filtering Oastler components that have a tolerance sensitive relation based on class type, as specified in the employed interdependency matrix (label format is: <preceding component> – <succeeding component>).***

Having inferred the tolerance sensitive relations and added them to the BIM, a range of rich queries (or "filters") can be applied. Figure 15 illustrates four such examples: (1) finding components that have a tolerance sensitive relationship with a given specific component (e.g., the illustrated fin). (2-4) finding pairs of components that have a tolerance sensitive relation of a specific type (parallelism, position, flatness) where the preceding component is on a given building story (i.e., floor 3.0 in the example).

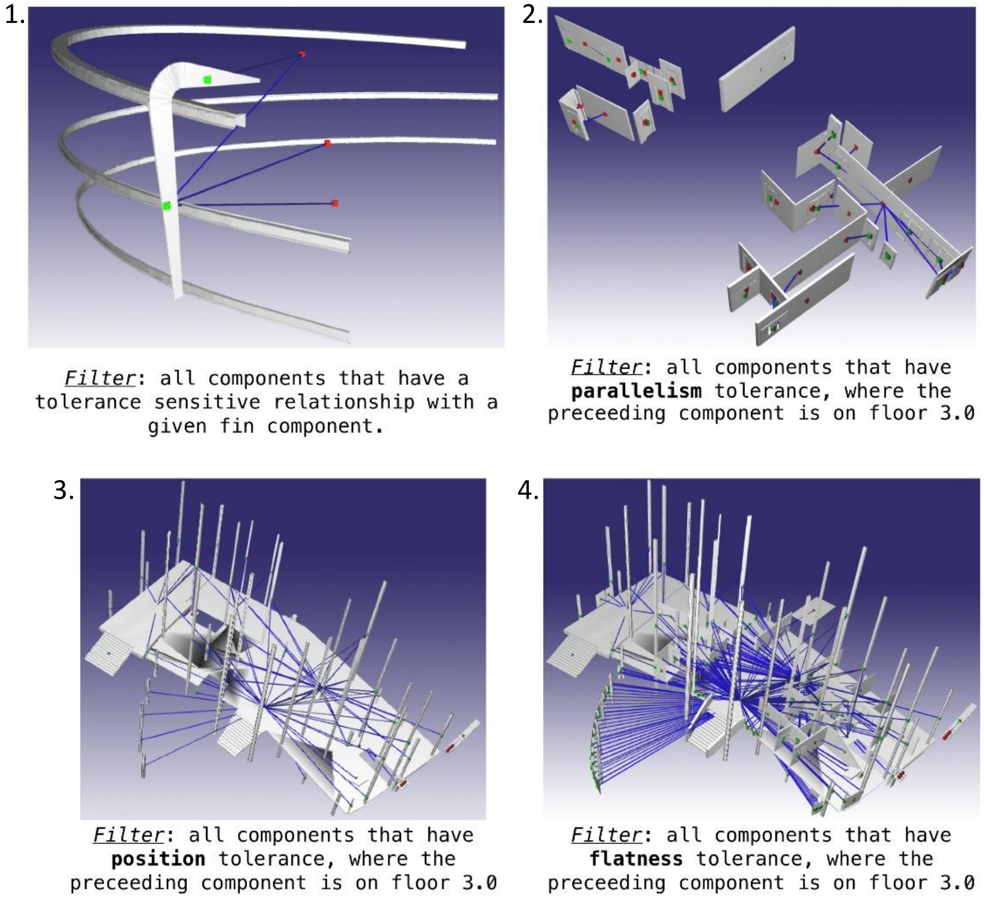


***Figure 15: Four filter examples that demonstrate the versatility of ToleranceDM.***

In both the "Simple Modular" and "Oastler", using the mesh bounding boxes resulted in some components not having contact with any other component. In such cases, the contact graph is not complete, and thus not all components are reachable from every other component through the contact graph, as illustrated in Figure 16. Methods such as Delaunay triangulation can be used to find minimum distances between components such that the contact graph is complete.

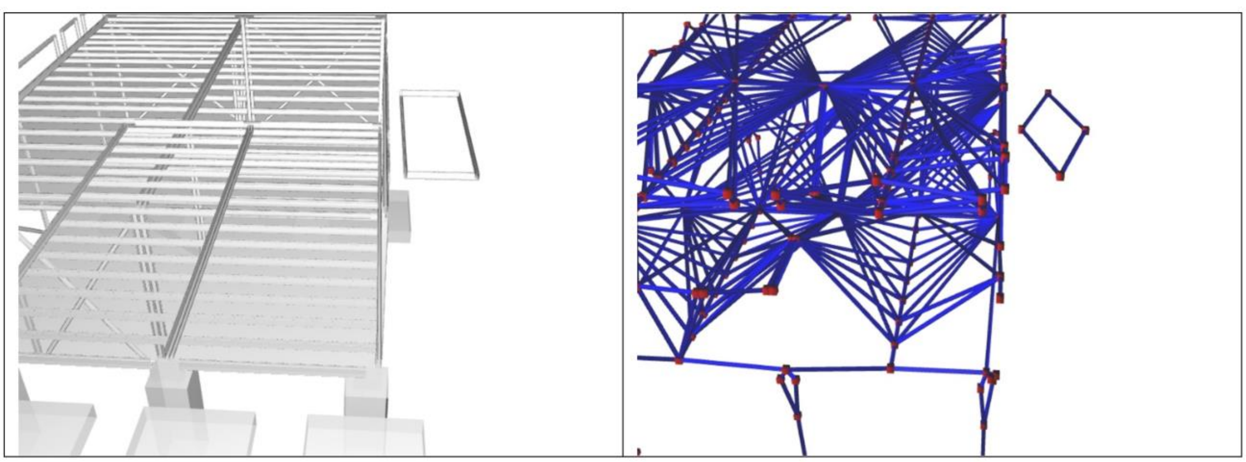

***Figure 16: Example illustrating "float freely" component groups.***

The computational runtimes for three derivation and analysis tasks are presented in Table 5. The longest task is parsing the BIM to derive the mesh geometries. Inferring all 13,000 contact relations of the Oastler completed within 1 minute, which is fast enough to be practical for construction planning. Column 3 entries indicate runtimes for deductive reasoning, i.e., compute the consequences of inference rules. Column 4 entries indicate runtimes for abductive reasoning, i.e., check compatibility of these consequences with domain constraints and amend inference rules if necessary. The validity check when applied to the (non-trivial) buffered bounding box case of the Oastler finishes within 21 seconds and is again fast enough to scale to large BIMs.

***Table 5: Computational runtimes for BIM augmentation tasks performed on the demonstration BIMs.***

| BIM | Time taken to parse IFC geometries into 3D meshes (seconds, 2dp) | Time taken to infer contact relations (seconds, 2dp) | Time taken to check contact validity with buffered bounding boxes (seconds, 2dp) |
|---|---|---|---|
| Simple Modular | 3.00 s | 0.38 s | 0.03 s |
| Oastler | 83.00 s | 57.65 s | 20.14 s |

## 6.0 Conclusions and Future Work

The specification, control and overall management of tolerances is a vital component to project success in the construction industry. This is manifested in both the design and construction stages of a project:

- As BIMs undergo stages of refinement in the design process, objects' geometric and functional features are described by increasing levels of development (LODs). LODs define the confidence level of a BIM, i.e., how well a model reflects the design intent. Any deviation should be subject to a tolerance check in order to validate or abstract the results of uncertain and indefinite information contained in BIMs. A holistic approach to manage and document tolerances is therefore highly valuable (yet currently elusive) to explicate requirements about BIMs with specific LODs, as it provides a common understanding of how much a BIM can be trusted.
- During the construction stage, dimensional variability associated with the form, position and orientation of components is unavoidable. If not properly understood, controlled or managed, this variability can propagate to cause expensive and time-consuming tolerance issues (predominately between the interfaces of components). Despite previous attempts to develop and address a holistic tolerance management framework, existing works to date have not been organized into a practical and automated process.

This paper is a novel contribution that bridges these gaps by developing a domain model for tolerance management (ToleranceDM) that can be universally applied across a wide range of projects. This domain model takes inputs in the form of a dependency structure matrix, which outlines the tolerance sensitive relations between components and parts of an overall construction assembly. Using spatial reasoning and an inference engine, tolerance sensitive relations are automatically identified, parsed, and enriched on the input BIM. As illustrated in the results of two case studies, an initial BIM is semantically enriched for tolerance management-focused visualization and querying. This enrichment is shown to be effective even when key connection details are not present in an initial BIM (i.e., LOD 300). ToleranceDM is shown to be effective and practical, through functional demonstration of a small BIM and a larger, more complex BIM.

### 6.1 Intelligently Inferring Contact Relations

As part of future work, a more comprehensive abduction (hypothetical reasoning) framework is being developed to intelligently infer a plausible set of contact relations as a basis for reasoning about tolerance sensitivity. The comprehensive abduction framework that is being developed is formulated as follows.

- Contact relations between each pair of components are the *abducibles* (candidate *hypotheses*). That is, if there are $N$ components, then there are $N^2$ component pairs, and because contact is symmetric and reflexive, then there are $N(N-1)/2$ distinct candidate hypotheses.
- Each *hypothesis* (that some pair of components have contact) must be *justified* by "evidence", i.e., the geometric information provided in the BIM (primarily distance and relative orientation) and the component class types. For example, in order for a contact relation to be justified, the distance between components must be within a specified threshold, and in some cases component surfaces must be parallel so that they can meet flush when moved together.
- A *scenario* is a set of hypothesized contact relations. Thus, the size of the scenario space is $O(2^N)$, i.e., exponential in the number of components.
- A *valid scenario* is one that satisfies all domain constraints, such as no "free floating" components.
- Scenarios are *ranked* by metrics, such as preferring scenarios that minimize the number of hypotheses.
- The *abduction task* is to find the highest-ranking valid scenarios.

A spatial reasoning extension of ASP is being utilized [51,52] to efficiently prune the search space of valid hypotheses. This is achieved by defining a series of increasingly accurate conditions that are necessary (but not sufficient) for meshes to satisfy distance and orientation constraints. For example, the following spatial constraints are necessary for two meshes to be within a threshold distance $D$, and each successively increases in computational complexity and accuracy:

- The intersection of bounding boxes buffered by $D$.
- The intersection of component-aligned bounding boxes buffered by distance D.
- Distance to hyperplane of separation being less than $D/2$.

The domain constraint determining contact graph validity is also being refined. In particular, the "free floating" constraint is not sufficient – components need to be *supported* so that they are stable within the assembly. Being supported implies contact, but also involves other component-specific criteria, the evaluation of which may require physics and statics simulators and corresponding qualitative spatial rules, e.g., "a component that has contact with a slab from below is supported by that slab". The revised validity condition is thus that every component is transitively supported by a foundation component.

This leads to yet another aspect of future work which is the automated enrichment of BIM class types. In BIM standards such as IFC, certain types of component classes are not defined although they are critical to reasoning about tolerance. For example, a slab may be laid at the base of a design as the foundation, although the class of the component in the BIM is IfcSlab (and no such class as IfcFoundation currently exists). Thus, logical rules are needed to reason about whether a component is a particular subclass, such as foundations.

### 6.2 Expanding the Tolerance Domain Model

As illustrated in Figure 1, the purpose for the first version of ToleranceDM is to provide a way to semantically enrich BIM with tolerance management concepts. In this paper, we outline how such an enriched BIM could be queried, however future work will expand upon the use cases to explore the following: automated tolerance analysis, automated tolerance synthesis, visualization of project risks related to tolerances, interactive development of tolerance interdependency matrix, and construction progress monitoring while considering tolerances. The developed version of ToleranceDM will enable users to interactively edit the default tolerance interdependency matrix and adjust it to the peculiarities of each project. For progress monitoring, ToleranceDM will be developed further to assess the compliance of installed components with the specified tolerances by comparing as-built data (e.g., a point cloud) with BIM as well as comparing the as-built data over time. This function will be linked with 4D BIM in order to check whether the installed components comply with the specified tolerances when components in critical connections are installed. This is particularly important when considering building movement (i.e., geometric changes over time as a result of deflection, drying shrinkage, foundation movement). For example, cladding systems should be capable of incorporating deviations of the structural assembly due to its self-weight and other loads applied afterwards as well as deviations of workmanship when installing the cladding system. The developed version of ToleranceDM will assess whether deviations of the structural assembly and cladding system comply with the specified tolerances at the time of installation. Further, the development team will conduct a more comprehensive refinement process through structured interviews with domain experts across the industry. In addition, ToleranceDM will be further scrutinized and tested on several large-scale construction projects.


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## Appendix A: Tolerance Interdependency Matrix for Oastler Functional Demonstration

| | Foundation | Concrete slabs | | Columns | | | Roof | Staircase | | Curtain Wall | | | Balustrade | Parapets | Fins | Internal Partitions | | | Stone Cladding | Office Doors | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Top surface | Top surface | Bottom surface | Top | Middle | Bottom baseplate | Top surface | Top | Bottom | Top adjustable connection | Bottom | Right/left | Bottom | Bottom | Top/ bottom adjustable | Top slotted head track | Bottom slotted head track | Right/ left | Side slotted bracket | Left/ rights vertical jamb | Bottom horizonal jamb | Top horizontal jamb |
| | $C_1^1$ | $C_2^2$ | $C_3^2$ | $C_4^3$ | $C_5^3$ | $C_6^3$ | $C_7^4$ | $C_8^5$ | $C_9^5$ | $C_{10}^6$ | $C_{11}^6$ | $C_{12}^6$ | $C_{13}^7$ | $C_{14}^7$ | $C_{15}^8$ | $C_{16}^9$ | $C_{17}^9$ | $C_{18}^9$ | $C_{19}^{10}$ | $C_{20}^{11}$ | $C_{21}^{12}$ | $C_{22}^{12}$ |
| $C_1^1$ | | | F | | | PO | | | | | | | | | | | | | | | | |
| $C_2^2$ | | | | | | | | | PO, F | | F | | F | | | | F | | | | F | |
| $C_3^2$ | F | | | PE | | PO, F | | PO, F | | F | | | | | | F | | | | | | F |
| $C_4^3$ | | | PE | | | | PE | | | | | | | | | | | | | | | |
| $C_5^3$ | | | | | | | | | | | | PO, PA | | | PO | | | | | | | |
| $C_6^3$ | PO | | PO, F | | | | | | | | | | | | | | | | | | | |
| $C_7^4$ | | | | | | | | | | | | | | F | | | | | | | | |
| $C_8^5$ | | | PO, F | | | | | | | | | | | | | | | | | | | |
| $C_9^5$ | | PO, F | | | | | | | | | | | | | | | | | | | | |
| $C_{10}^6$ | | | F | | | | | | | | | | | | | | | | | | | |
| $C_{11}^6$ | | F | | | | | | | | | | | | | | | | | | | | |
| $C_{12}^6$ | | | | | PA, PO | | | | | | | | | | | | | | | | | |
| $C_{13}^7$ | | | | | | | | | | | | | | | | | | | | | | |
| $C_{14}^7$ | | | | | | | F | | | | | | | | | | | | | | | |
| $C_{15}^8$ | | | | | | | | | | | | | | | | | | | | | | |
| $C_{16}^9$ | | | | | | | | | | | | | | | | | | | | | | |
| $C_{17}^9$ | | | | | | | | | | | | | | | | | | | | | | |
| $C_{18}^9$ | | | | | | | | | | | | | | | | | | | | PA | | |
| $C_{19}^{10}$ | | | | | | | | | | | | | | | | | | | | | | |
| $C_{20}^{11}$ | | | | | | | | | | | | | | | | | | PA | | | | |
| $C_{21}^{12}$ | | | | | | | | | | | | | | | | | | | | | | |
| $C_{22}^{12}$ | | F | | | | | | | | | | | | | | | | | | | | |

F= Flatness tolerance, PA: Parallelism tolerance, PO: Position tolerance, PE: Perpendicularity tolerance.

Red = high risk, orange = med risk, green = low risk.

## Appendix B: ToleranceDM Specification in EXPRESS

```
SCHEMA ToleranceDM;

ENTITY TmsRelToleranceSensitiveConnects
 ABSTRACT SUPERTYPE OF(ONEOF(TmsRelSize, TmsRelForm, TmsRelOrientation, TmsRelLocation))
 SUBTYPE OF (IfcRelConnects);
  RelatingElement : IfcProduct;
  Provenance : OPTIONAL TmsProvenanceEnum;
  Risk : OPTIONAL TmsRiskCategory;
  Data : OPTIONAL IfcPropertySet;
END_ENTITY;

ENTITY TmsRelSize
 SUBTYPE OF (TmsRelToleranceSensitiveConnects);
END_ENTITY;

ENTITY TmsRelForm
 SUBTYPE OF (TmsRelToleranceSensitiveConnects);
  Form : TmsFormEnum;
END_ENTITY;

ENTITY TmsRelOrientation
 SUBTYPE OF (TmsRelToleranceSensitiveConnects);
  Orientation : TmsOrientationEnum;
  RelatedElement : OPTIONAL IfcProduct;
 WHERE
  ParallelismHasDatum : Orientation <> TmsOrientationEnum.PARALLELISM OR EXISTS(RelatedElement)
END_ENTITY;

ENTITY TmsRelLocation
 SUBTYPE OF (TmsRelToleranceSensitiveConnects);
  RelatedElement : OPTIONAL IfcProduct;
END_ENTITY;

ENTITY TmsRiskCategory
 SUBTYPE OF (IfcRoot);
  RiskType : TmsRiskTypeEnum;
  Severity : TmsRatingEnum;
  Likelihood : TmsRatingEnum;
END_ENTITY;

TYPE TmsProvenanceEnum = ENUMERATION OF (
 NORMAL,
 SPECIAL,
 SPECIFIC);
END_TYPE;

TYPE TmsRiskTypeEnum = ENUMERATION OF (
 STRUCTUALSAFETY,
 CONSTRUCTABILITY,
 AESTHETIC,
 FUNCTIONALITY);
END_TYPE;

TYPE TmsRatingEnum = ENUMERATION OF (
 LOW,
 MEDIUM,
 HIGH,
 UNSPECIFIED);
END_TYPE;
```

```
TYPE TmsFormEnum = ENUMERATION OF (
 STRAIGHTNESS,
 FLATNESS);
END_TYPE;

TYPE TmsOrientationEnum = ENUMERATION OF (
 PERPENDICULARITY,
 PARALLELISM);
END_TYPE;

END_SCHEMA;
```