Activity Based Easy Learning Of PushDown Automata

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Abstract—Teaching is a skill. It is significant to be aware or enquires how to express this skill. Teacher must have fully gripped upon his art of teaching. He must know the psychology, mental and emotional distorting elements of students to deal with in all situations accordingly. Learning disability is not a big difficulty or harder-ship and can achieve easily by developing interest and motivating students by introducing some new updated tools and methodologies like Thoth, SELFA, FLUTE, JFLAP, FLAP, Java Finite Automata, Deus Ex Machina (DEM) and homework exercises for practicing by hand, workshops, oral assessments, quizzes, group sharing, group assessment, clustering and feedback respectively that are explain below. This article provides different ways of teaching PDA and makes students learning process easy. Furthermore, this article also clear the conceptual model of PDA and enhance the ability to design PDA machine conveniently.

Keywords: Push down Automata (PDA), Theory of computation (ToC), Formal language, Learning tools, Methodology, DEM, Design, JFLAP, Teacher, Students

I. INTRODUCTION

Theoretical automata is a formal language, which has important roles in many areas particularly in theory of computation, artificial intelligence and compiler Constructions. PDA Teaching is a challenging because students face difficulty in learning pushdown automata because subject of theory of automata is tedious or tiresome and mathematical in nature. Graduated students of computer science have no strong background of mathematics so they are not motivated to take this course of automata. Students track to be archaic that cannot associate their course and computer application. These things cause lack of interest and disappointment in mostly students, which leads to increase the failure or dropout rate. This course mostly introduced without any practical work. Conventional techniques use that leads to integration of lecture with tutorial amalgam. Chalk and board or white board approach is used to teach the students. First of all instructor clear the terminologies regarding pushdown automata once the conceptual model and its functionality is understand by students then recognize context free grammar and design pushdown automata machine easily. For easy learning instructor use different tools and methodologies for practical work. Practical work always develops interest in students. This article encourages the students for learning PDA. Presented tools and strategies decrease the complexity that students faced in PDA learning. Different software tools are used that simulate theory and practice.

II. RELATED WORK

M. Vijaya and Karibasappa determine the hierarchy structure for the formal language and pushdown automata. According to them teaching should be activity based so the students take part and show more interest in the class lecture. The burden and stress of examination also reduced if they done their activities related to PDA and formal languages.

Mikel and Montserrat explain that JFLAP is visual software for teaching purpose of Automata. Dana Angluin introduced the algorithm that everyone visualized learning process step by step.

Gabriel, Celso, Ronaldo and Rogerio proposed the method of simulator based work for the students. They develop a simulator, which is based on multi-formalism which more helps the student to understand the abstract concepts in an easy way due to visualization.

Furthermore S. A. ALI++, S. SOOMRO, A. G. MEMON* A. BAQI** describe about the disabilities (means students don’t have mathematics background or some lack of other subject knowledge) of the student which can be handled by motivating the students, creating the emotional class room environment and divide the students in groups for better learning.

There are a number of approaches that may provide assistance in designing emerging solutions for the rising challenges in designing smart as well as autonomous management systems. Fuzzy logic design [7], Machine learning [9-11,13, 16-20, 22, 24,27,28,29,30], soft computing

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Particle Swarm Optimization (PSO), Genetic Algorithm, and round robin, and blockchain technology are some of the approaches that are being used while employing and constructing a number of smart, and autonomous frameworks.

III. TOOLS

A. Thoth

This tool is used to learn formal language of automata and PDA is design by using simple operation of mouse clicking. Thoth adds some new functionality like improve the graphic interface. We can undo, redo drawing operations and zoom in and out as well.

B. FLUTE

It is stand for formal language and automata environment. It is an intelligent tutoring tool that is used for simply design pushdown automata and makes learning easy.

C. JFLAP

It is stands for Java Formal Languages and Automata Package. It is a free of cost and interactive educational tool that is used to design and control difficulties of PDA in ToC course. We can draw diagrams easily and quickly. States can be move, edit and label by using this tool.

D. Deus Ex Machina (DEM)

It is a simulator, which is design and running different kinds of automata like FA states, Pushdown Automata and Turing Machine etc.

E. Automata Tutor v3

This is an update version, which is used for generating automated grading and feedback for new variety of problems. User can create RE, CFG, PDA and TM corresponding to the given description. This version is also used for proving non-regularity of languages by using pumping lamma, R.E, context free grammar, PDA and Turing Machine. Conversion is possible between such models.

F. Automata online tool

It supports the students to comprehend important notation of PDA in ToC. This tool provides feedback when students give wrong answers. It helps instructor in grading H.W assignments for large classes.

IV. METHODOLOGY

Selecting right methodology is an art of a teacher if he chooses it correctly then him beautifully, simply and easily explains his ideas and conveys the lecture, which clears the all concepts of PDA. Demonstrated activity in the classroom on pushdown automata make the students active and focused. Mostly students ignore the tedious work by hand but Practicing by hand of different PDA design and observing step by step algorithm make the students hard working and consistent in nature.

Teacher should give the homework-based exercise for practice, which contains different PDA problems. It helps the students to understand, think about problems, enhance the knowledge and design various PDA machines by searching different websites and literatures.

Furthermore, teacher should arrange the workshops for effective learning of different tools, which are used in designing of automata.

In addition, quizzes and oral assessment of Pushdown automata is also an effective technique to become students active during class learning.

Group sharing and discussing with pairs is a brilliant technique that generate new ideas and enhance the quality of PDA learning. Conducting presentation increases the confidence of students.

“Share and care” develop interest in students that aid to learn such formal languages of PDA, it contribute to how research work do which cause the expansion of Turing Machine.

V. FORMAL DEFINITION OF PDA

Formally “Pushdown automata” defined as $Q \times (\Sigma \cup \{\varepsilon\}) \times \Gamma \times Q \times \Gamma^*$

These 7 tuples are following below;

It is formally described by 7-tuple $(Q, \Sigma, \Gamma, \delta, q_0, I, F)$:

- $Q$: It is a finite number of states
- $\Sigma$: It represent alphabet of input
- $\Gamma$: Stack symbol that can push and pop at the top of the stack
- $\delta$: Function of transition, $Q \times (\Sigma \cup \{\varepsilon\}) \times \Gamma \times Q \times \Gamma^*$
- $q_0$: Shows initial state ($q_0 \in Q$).
- $Z_0$: It represents start symbol, which is in $\Gamma$. It can be denoted as $\$ symbol instead of $Z_0$
- $F$: Represent accepting states of set ($F \in Q$)

VI. CONCEPTUAL MODEL OF PUSHDOWN AUTOMATA

PDA has three basic components as following below;

A. An Input Tape

New symbol added at the top of the input tape. It divided into infinite cell each contains symbols ($\Sigma$). The head of the input
is read only and one symbol can scan left to right at a time.

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**C. A Stack With Infinite Size**

In stack one end can only put and remove items. It has an infinite size. In “PDA” stack temporarily stored items. It works in LIFO manner.

The head of stack scans top most symbol lies in the stack. It performs two main operations:

a) **Push Operation**

- Added new symbol on the stack’s top.

b) **Pop Operation**

- Top symbol is read and removed from the stack.

---

**B. A Control Unit**

The FC (finite control) has some pointer which points the currently read symbol.

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**VII. GRAPHICAL NOTATION (PDA)**

It can be represented graphically following below as

---

**VIII. INSTANTANEOUS DESCRIPTION (ID)**

It is PDA informal notation that represented with triplet (q, w, $). It describes how to compute an input string then make a decision after that string acceptance and rejection will happen. Triplet (q, w, a) define as where:

- q: Determine the present state.
- w: determines unread input.
- a: determines content of the stack at the top from left to right.

**IX. TURNSTILE NOTATION (TN)**

"Turnstile notation" is a mechanism that ex. TN is an explanation of the ‘Instantaneous Description’ (ID) and connects pairs of ID’s. TN represented by the symbol “$-”also
called right tack. It defines the transition process and determined how one move or many moves occur in PDA.

a. This symbol \( \vdash \) describes turnstile notation and single move.
b. This symbol \( \vdash^* \) describes zero or multiple moves of the PDA.

Consider a PDA which contains seven tuples “\( Q, \Sigma, \Gamma, q_0, \delta, I, F \)”, then mathematical transition of ‘TN’ can be described as following example:

**Before TN transition**
\( (p, aw, T) \vdash (q, w, \alpha) \)

**After TN transition**
\( (p, w, T\beta) \vdash (q, w, \alpha\beta) \)

Transition taking from state ‘\( p \)’ to state ‘\( q \)’, the input symbol ‘\( a \)’ consumed and ‘\( T \)’ top of the stack is replaced by a new string ‘\( a \)’.

Let’s now construct ‘Instantaneous Description’ (ID).

X. PDA DESIGN USING JFLAP

Example 1: Design a PDA for language that accepts \( L= \{c^n d^n | n > 0\} \) and give ID for a string \( w= cccddd \)

Solution: In the above language, \( n \) number of \( c \)’s should be followed by \( n \) number of \( d \)’s. So, we will use a simple logic, If we read one ‘\( c \)’, we will push one ‘\( c \)’ onto the stack again loop read ‘\( c \)’ and push ‘\( c \)’ into the stack until all \( c \)’s ends. When we read ‘\( d \)’ for every single ‘\( d \)’ only one ‘\( c \)’ should get popped until all \( d \)’s popped out from the stack.

\[ \delta(q_0, c, \varepsilon) = (q_1, Z_0) \quad (\text{there is no symbol to pop}) \]
\[ \delta(q_1, d, c) = (q_2, c) \]
\[ \delta(q_2, \varepsilon, Z_0) = (q_3, \varepsilon) \]

Now this PDA will simulate the input string ” \( w= cccddd \) “. \( \delta(q_0, cccddd, \varepsilon) \vdash (q_1, cccddd, Z_0) \)

\[ \varepsilon (q_1, cccddd, cZ_0) \quad (\text{push}) \]
\[ \varepsilon (q_1, cddd, ccZ_0) \quad (\text{push}) \]
\[ \varepsilon (q_1, ddd, cccZ_0) \quad (\text{push}) \]
\[ \varepsilon (q_2, dd, ccZ_0) \quad (\text{pop}) \]
\[ \varepsilon (q_2, d, cZ_0) \quad (\text{pop}) \]
\[ \varepsilon (q_2, \varepsilon, Z_0) \quad (\text{pop}) \]
\[ \varepsilon (q_3, \varepsilon, \varepsilon) \rightarrow \text{Accepted} \]

Example 2: Define the PDA for the language \{\( b^n a^n | n >=0 \} \) using by final state.

Solution:

\[ \delta(q_0, b, Z_0) = (q_1, BZ_0) \]
\[ \delta ( q_1, b, B ) = ( q_1, BB ) \]

\[ \delta ( q_1, a, B ) = ( q_2, \epsilon ) \]

\[ \delta ( q_2, a, B ) = ( q_2, \epsilon ) \]

Here we explain how PDA works:

In the above language, number of b’s should be followed by number of a’s, both ‘a’ and ‘b’ must be equal. Hence, we will use simple logic that if we read single ‘b’, we will push one B’s into the top of the stack again loop read ‘b’ and push ‘B’ into the stack until all b’s ends. As we read ‘a’ then for every single ‘a’ only one ‘B’ should get popped from the stack until all B’s popped from the stack.

In the above transition row 0, we push initial symbol of the stack Z0. This symbol shows that there is no another symbol available in the stack. We use epsilon symbol ‘\( \epsilon \)’ instead of lambda both shows the empty or null symbols so don’t be get confused.

This \( \delta \) represents the transition state.

\( q_0 \) is initial state

Moving from initial state \((q_0, b, Z_0)\) to \(q_2\), we push single B on to the stack. Remaining at the state \(q_2\) all the B pushed on to the stack. Now every B popped from the stack correspondence to the every single ‘a’. At the final state all B popped and stack will be empty. This shows that string accepted by final state.

**Table 1**

<table>
<thead>
<tr>
<th>Row</th>
<th>State</th>
<th>Input</th>
<th>( \delta ) (Transition Function) used</th>
<th>State Leftmost symbol show state</th>
<th>Stack after move</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>( q_0 )</td>
<td>bbbaa</td>
<td>( \delta ) ((q_0, b, Z_0) = ((q_1, BZ_0)))</td>
<td>B Z_0</td>
<td>Z_0</td>
</tr>
<tr>
<td>1</td>
<td>( q_0 )</td>
<td>bbbaa</td>
<td>( \delta ) ((q_1, b, B) = ((q_1, BB)))</td>
<td>BB Z_0</td>
<td>q_1</td>
</tr>
<tr>
<td>2</td>
<td>( q_1 )</td>
<td>bbbaa</td>
<td>( \delta ) ((q_1, a, B) = ((q_2, \epsilon)))</td>
<td>BBB Z_0</td>
<td>q_1</td>
</tr>
<tr>
<td>3</td>
<td>( q_1 )</td>
<td>bbbaa</td>
<td>( \delta ) ((q_1, a, B) = ((q_2, \epsilon)))</td>
<td>BBB Z_0</td>
<td>q_1</td>
</tr>
<tr>
<td>4</td>
<td>( q_1 )</td>
<td>bbbaa</td>
<td>( \delta ) ((q_1, a, B) = ((q_2, \epsilon)))</td>
<td>BBB Z_0</td>
<td>q_1</td>
</tr>
<tr>
<td>5</td>
<td>( q_2 )</td>
<td>bbbaa</td>
<td>( \delta ) ((q_2, a, B) = ((q_2, \epsilon)))</td>
<td>BB Z_0</td>
<td>q_2</td>
</tr>
<tr>
<td>6</td>
<td>( q_2 )</td>
<td>bbbaa</td>
<td>( \delta ) ((q_2, a, B) = ((q_2, \epsilon)))</td>
<td>BB Z_0</td>
<td>q_2</td>
</tr>
<tr>
<td>7</td>
<td>( q_2 )</td>
<td>( \epsilon )</td>
<td>( \delta ) ((q_2, \epsilon, Z_0) = ((q_3, \epsilon)))</td>
<td>Z_0</td>
<td>q_3</td>
</tr>
</tbody>
</table>

\[ \delta ( q_2, \epsilon, Z_0 ) = ( q_3, \epsilon ) \]

**Case 1:** If input symbol & stack symbol is same

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>1</th>
<th>Z</th>
</tr>
</thead>
</table>

**Case 2:** When our guess is wrong.

Here we explain how PDA works:

In the above language, number of b’s should be followed by number of a’s, both ‘a’ and ‘b’ must be equal. Hence, we will use simple logic that if we read single ‘b’, we will push one B’s into the top of the stack again loop read ‘b’ and push ‘B’ into the stack until all b’s ends. As we read ‘a’ then for every single ‘a’ only one ‘B’ should get popped from the stack until all B’s popped from the stack.

In the above transition row 0, we push initial symbol of the stack Z0. This symbol shows that there is no another symbol available in the stack. We use epsilon symbol ‘\( \epsilon \)’ instead of lambda both shows the empty or null symbols so don’t be get confused.

This \( \delta \) represents the transition state.

\( q_0 \) is initial state

Moving from initial state \((q_0, b, Z_0)\) to \(q_2\), we push single B on to the stack. Remaining at the state \(q_2\) all the B pushed on to the stack. Now every B popped from the stack correspondence to the every single ‘a’. At the final state all B popped and stack will be empty. This shows that string accepted by final state.

**A. Non-Deterministic Pushdown Automata (NDPA)**

It accepts one or more moves on state of an input symbol and stacks symbols.

Construct a PDA for a ‘L’ language.

\[
L = \{ w^2 \mid w = \{ 0, 1 \}^* \}
\]

\[
W = 0110
\]

**Example 1:** 010010

\[
(q_0, 010101, 0z)
\]

\[
(q_0, 010010, 0z)
\]

\[
(q_0, 010010, 10z)
\]

\[
(q_0, 010010, 10z)
\]

\[
(q_0, 010010, 11z)
\]

\[
(q_0, 010010, 11z)
\]

\[
(q_0, 010010, 11z)
\]

The string is accepted.

**Example 2:** Let’s considered following “NPDA” as:

**Fig 8.**

**Fig 9.**

\[
(q_0, e, D) = \{(q_1, CD), (q_3, \lambda)\}
\]

\[
(q_0, \lambda, D) = \{(q_3, \lambda)\}
\]

\[
(q_1, e, D) = \{(q_1, CD)\}
\]

\[
(q_1, e, C) = \{(q_1, \lambda)\}
\]

\[
(q_2, e, C) = \{(q_2, \lambda)\}
\]
We can recognize the string ‘eeff’ by the following sequence of moves:

\[
\delta (q_2, \lambda, D) = (q_3, \lambda)
\]

Since \( q_3 \in F \), the string is accepted.

XI. CONCLUSION AND FUTURE WORK

Flow of teaching discipline has made students to understand the concepts and relate to the real time application easily. PDA learning has different issues for teacher first indicating student’s dropout rate due to the demotivation and doesn’t have mathematical background. This paper has presented different methodologies and tools for learning PDA in easy way which motivate and develop interest in students. Proposed strategies apply to small groups of students, conducting workshops for learning software tools that help to design PDA machine. However, learning about many tools also increases the burden of work. Therefore, students should learn few tools according to the requirements and their interest. Sharing material, students evaluate each other in cluster group that encouraged, increased their confidence and interest level, which motivate them to learn pushdown automata.

In future, we expect the updating new version of tools, which expand the Pushdown automata applications that relate the PDA with real world and artificial intelligence.

REFERENCES


