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Secondary Spectrum Allocation Framework via Concurrent Auctions for 5G and Beyond Networks

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Abstract Due to the dramatic increase of spectrum demand, efficient usage of the limited spectrum resources has become a crucial issue for the next-generation wireless networks. Auction-based spectrum trading, utilization and pricing have many promising features and have proven to be a fair and consistent way of secondary spectrum trading and management. In this paper, we present a mathematical approach to the future spectrum market where multiple buyers (secondary network operator, SNO) compete to gain spectrum resources through a number of auctions from multiple sellers (primary network operator, PNO). Through static and dynamic auctions, the secondary network operators borrow underutilized licensed spectrum resources from primary operators either through predefined contracts or through instantaneous contracts. Our main focus is on the optimal choice of the secondary operator, contiguous spectrum resource to maintain the quality and utilization history based fair allocation of the spectrum resources through auctions controlled by the third party spectrum regulators (SR), which has not been addressed previously. We first develop a matching problem to identify the most suitable auctions for secondary operators. A price-based optimal number of auctions and a utility-based ranking of the optimal auctions to be bid by the secondary operators are proposed, where the secondary operator maximizes the net utility surplus (NUS). The win or lose, pricing and allocation of spectrum resources are determined by a proposed Vickery-type mechanism. Finally, we provide simulation results to evaluate the performance of the proposed auction mechanism.

Keywords Auction mechanism · Optimal spectrum lease · Spectrum management · Spectrum market

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1 Introduction

With the vast expansion in the scale of wireless networks and types of network services marked in recent years, the available radio resources are increasingly becoming scarce than ever before. Despite the decentralization of spectrum allocation and management systems, increase in the efficiency of spectrum utilization through spectrum liberalization and dynamization, expansion in the range of usable radio spectrum, and advent in physical layer technologies (e.g., multiple-access, etc), there still exists a substantial scope to further explore all the horizons in order to meet the anticipated stringent spectrum demands of ultra massively connected wireless networks in beyond 5$^{\text{th}}$ generation (B5G) and 6$^{\text{th}}$ generation (6G) wireless networks.

1.1 Key challenges in B5G & 6G networks

The first standardization effort of 5$^{\text{th}}$ Generation (5G) wireless networks has recently commenced through Release-15 of 3$^{\text{rd}}$ Generation Partnership Project (3GPP) [50]. Various new revolutionary technologies extending the provisions of several new types of innovative network services are being offered in the release. Along with the offered numerous fundamental advantages, an explosive increase in the number of connected wireless devices is also anticipated in the coming years. The global number of machine type subscribers and mobile users are projected to reach 97bn and 17.1bn [31] by the year 2030, respectively. This massive increase in the scale of wireless networks is expected to reinstate the conventional wireless communications challenges and also lead to various diverse types of new challenges; e.g., this may lead to rapidly reaching the limit of network capacity and scarcity of available radio resources, etc [44]. In this regard, researchers around the world have started exploring new usable radio spectrum, developing more efficient spectrum utilization and management schemes, and finding solutions to the subsequent challenges that may emerge in B5G and 6G era, see e.g., [17, 41, 49]. For a comprehensive overview of resource management in 4G, 5G and beyond networks, see [5, 40] and references therein.

1.2 Dynamic spectrum management in B5G/6G networks

The scarcity of available radio spectrum caused by the static frequency allocation policies and fragmented allocation across different services could be addressed by utilizing suitable dynamic spectrum sharing techniques [48]. There mainly exist three types of dynamic spectrum sharing models, namely, spectrum-commons model, shared-used model and exclusive-use model, which has their own advantages and disadvantages [28,44]. The spectrum-commons model, which provides equal rights to all the secondary/unlicensed users, is mainly suitable for dynamic spectrum sharing of the unlicensed bands. Whereas, the shared-used model enables the secondary users to use the underutilized or vacant spectrum either in an interference avoidance manner or in an opportunistic way with the help of various cognitive radio techniques [47]. On the other hand, the exclusive-use model enables secondary users to obtain exclusive spectrum usage rights from the primary system either by giving a cooperation reward or by purchasing a portion of the spectrum.

Besides the decentralization introduced in the radio spectrum management schemes, there exist various diverse causes that restrict the adequate proactive utilization of secondary radio spectrum bands, e.g., the variations in spatial and temporal statistics of resource demands, etc. The liberalization and dynamization of spectrum licensing and spectrum management, respectively, have significantly contributed towards the improvement in the efficiency of radio resource utilization over the years. The spectrum liberalization refers to providing exclusive usage right of spectrum resources to more than one commercial and government departments [20, 26, 35, 43]. The dynamic spectrum access (DSA) allows the unlicensed users (i.e., secondary users) along with licensed users (primary users) to use the spectrum resources opportunistically has brought some distinct features to increase the efficiency of spectrum usage and reduce the scarcity of the spectrum resource. Nevertheless, there is still a substantial scope to further improve DSA through revolutionizing the spectrum policy as well as technologies to deliver sustainable spectrum resource management solutions for the emerging era of beyond 5G (B5G) and 6G wireless networks.

1.3 Related works

Spectrum management for secondary users via DSA is a well-studied area and can be performed broadly in two ways: secondary spectrum sharing [2, 7, 10] and secondary spectrum trading [11, 26, 30, 38]. However, secondary spectrum trading can be sub-categorized into three categories– direct trading, brokerage and auction [42, 56]. The auction-based secondary spectrum borrowing has been a promising technique to regulate and coordinate spectrum resources [15, 25, 32]. It allows the spectrum owners (operators) to earn revenue by the reuse of the spectrum resource to secondary users whenever the resource is idle. This unique feature would increase spectrum efficiency and social welfare in future wireless communication environments [4, 12, 51, 52]. Market-based secondary spectrum trading through auction-mechanism has been studied by many researchers [15, 25, 56]. Nonetheless, we have witnessed only a few countries like the USA, UK, Australia, New Zealand have adopted the spectrum trading strategies. It is well anticipated that this approach is going to be the key when the next generation (6G and beyond) of communication will be introduced.

Spectrum management via auctions involves a series of important inter-related functionalities, which are auction choice and selection, contract, pricing, competition, policy, allocation, etc. [34, 37, 55, 57]. There are many diverse issues and extensive literature is available in each of the areas. Auction selection is one of the key issues from buyers (PNOs) point of view which is addressed by many researchers [3, 22, 26, 54]. For example, the work presented in [26] proposed a general integer programming method to select suitable auctions. A game-theoretic model was proposed in [22], showcasing potential. In [54], a distributed frequency reuse, in a form of the graph-theoretic model was put forward. Many researchers studied auction pricing and some recent studies include [33, 58]. For more pricing models, see the survey paper by [28] and references therein. The key aspect of resource trading by auctions is the revealing of the concealed cost at the pre-bidding stage which is addressed in [16] and fair payment and winning prices in [18].

Extensive theoretical results are also available in the literature on auction design, statistical properties, optimality, truthfulness and stability proof. A host of other papers dealt with auction design and resource allocation, those include [9, 19, 26, 36, 56]. In a study of secondary spectrum trading presented in [56], its authors compared direct trading, auctions and a brokerage mechanism for secondary spectrum use under a market-changing environment. A more in-depth online and offline auction mechanism for secondary spectrum utilization has been studied by [25, 26], where [26] proposed an auction trading framework through economic incentives to improve spectrum utilization efficiency by spatial reuse of spectrum under deterministic and stochastic information.

As our focus is to find the most suitable auctions for primary users based on the guaranteed quality of service (QoS) using contiguous bandwidth, minimizing the cost of the lease and gaining a higher utility of spectrum usage, we highlight the relevant works on these aspects only.

1.3.1 Auction matching with contiguous bandwidth, cost minimization and utility maximization

Auction matching has been a well-studied area, particularly for common goods. Extensive methodological techniques have been used to address the issue. In the literature of auction matching, *stable matching* and *auction-based spectrum allocation* have been extensively studied and used in many fields [24]. The stable matching problem deals with finding a stable match between two equally sized sets of elements given an ordering of preferences for each element [14]. Deferred acceptance algorithm (DAA) and many other variants are available to implement the stable matching problem [45]. The stable matching was used for resource allocation in computer science. For instance, [24] proposed an algorithm to match heterogeneous sized jobs to virtual machines. It has also been used in wireless communications. To match users in device-to-device to cellular users for resource sharing [27] and to associate users to small cells [46], stable matching frameworks have been implemented. The double auction mechanism, on the other hand, is an important spectrum allocation paradigm for dynamic spectrum access. In a double auction mechanism, a spectrum regulator (third-party auctioneer) executes auction mechanisms to decide the spectrum allocation based on buyers' bids and the asks of sellers, which can also be referred to as an auction matching process. With such matching enforced by the spectrum regulator, the primary objective is usually to maximize the utility, revenue or truthfulness. [59] first proposed a truthful double auction spectrum allocation. Several truthful auction mechanisms were designed to deal with spectrum heterogeneity [21, 39], revenue maximization [6, 29], utility maximization [25, 26].

The main goal of spectrum auction is to achieve spectrum reuse, gain higher economic robustness and maximize social welfare. Auction participants are, in general, selfish and rational individuals who seek to obtain the optimal auction strategy to maximize their profit/revenue or minimize cost. The key aspect of auction design is the economic-robustness, which includes individual rationality, truthfulness, budget balance, social welfare, spectrum efficiency, seller and buyers' happiness, collusion and privacy preservation [13]. Over the last two decades, numerous *auction-based spectrum allocation mechanism* have been proposed: forward spectrum auction, combinatorial spectrum auction, homogeneous double spectrum auction, heterogeneous double spectrum auction and online spectrum auction [13, 56]. Several security threats have been identified in spectrum auction including bidding values, bidder identities, bid rankings, locations, and requested time slots.

Despite numerous investigations, there are many challenges yet to be tackled in auction mechanism due to interference constraint and corresponding spectrum reusability [23]. For example, carrier aggregation is used in LTE-Advanced to increase the bandwidth and to increase the bitrate. Each aggregated carrier can be referred to as a component carrier (CC). The component carrier may have several bandwidths, for instance, 1.4, 3, 5, 10, 15 or 20 MHz and a maximum of five component carriers can be aggregated simultaneously with an aggregated bandwidth of 100 MHz [50]. Multi-carrier high-speed downlink packet access (HSDPA) introduced in 3GPP with Dual-Cell HSDPA (DC-HSDPA) increases the transmission bandwidth and consequently achieves the maximum data rate. In multi-carrier downlink schemes, an adjacent multi-carrier operation is defined for operation in the contiguous spectrum and the nominal carrier spacing is 5 MHz, but other values of the channel spacing are also feasible [50]. This carrier aggregation, including contiguous and non-contiguous spectrum, is an important technique to increase bandwidth for transmission. This is essential for achieving high throughput and reliability. In dynamic spectrum allocation and the auctioning design, the availability of these bandwidths play an important role. For bidders with low channel requirements, it is intuitive to bid for channels that are without adjacent available carriers. On the other hand, it is preferable for bidders with higher bandwidth requirements to bid for carriers that can provide contiguous aggregation, up to the requirement. Non-contiguous carrier aggregation should only be considered when other options are exhausted, as a final recourse. According to the 3GPP specification [1], non-contiguous aggregation is limited to the bandwidth gap between the available carriers, due to the absolute and relative power tolerance. In addition, in 5G-NR FR2, the aggregation is limited to 8 channels with various configurations. We take all these considerations into our framework design to ensure maximum efficiency and compliance in this respect.

To address the auction selection for contiguous bandwidth in B5G/6G, there is no methodological results to the best of our knowledge when secondary operators (buyers) seek to find auctions considering the contiguity of concurrent auctions and to select the optimal number of auctions minimizing the bidding price. Moreover, it is not clear from the literature - which auctions to choose when a bidder wins more auctions than the bidder needs.

1.4 Main issues and contributions

Spectrum management is a crucial and challenging task for ensuring the quality of service in the next-generation wireless communication environment. In-depth efforts are required for the efficient operations of bandwidth management for the network operators despite the new ultra-modern technologies to be involved in the future communication systems. Although many studies have highlighted different issues including economic analysis (incentive or profit), pricing, contracts, market equilibrium, to the best of our knowledge no study deals with

- spectrum management based on contiguous and non-contiguous carrier aggregation that allows high throughput and ultra-reliability of bandwidth,
- utilization history to increase spectral efficiency,
- minimization of the bidding price under budget constraints, and
- ranking and selection of optimal auctions based on utility surplus value.

Our proposed method tackles these issues simultaneously and provides a fair deal of spectrum trading maintaining the quality of spectrum gain when multiple buyers and sellers are competing in the market. Our proposed framework simultaneously answers several key questions: (a) an SNO point of view– how to select the best set of auctions to bid considering the required amount and quality (measured in contiguity of spectrum bandwidth), how to minimize the cost of revealing the set of auctions and how to select the most suitable to auctions for gaining resources by minimizing total cost; (b) a PNO point of view– how to make a fair distribution of unused spectrum resources among the SNOs who require the most (measured in SNOs utilization history); and (c) a spectrum regulator perspective– how to allocate resources from PNOs to SNOs by setting a fair allocation with fair price deals.

Our main contributions in this paper are:

- we propose a new pragmatic spectrum trading model where non-contiguous channel aggregation for each bidder is used to achieve its aggregate demand to maintain the quality of the spectrum resources and information on the utilization history extracted from past allocations to bidders is used for the fair allocation,
- we formulate the problem of finding optimal total reserve cost for adversaries bidding on auctions while minimizing the winning costs to bidders, and
- we provide a utility gain framework which can be used by bidders to chose the optimal auctions that maximize monetary return after winning a set of auctions.

The remainder of the paper is organized as follows. In Section 2, we introduce the system model. In Section 3 we present the problem formulation and solution. Numerical investigations are shown in Section 4. Discussions and conclusion are given in Section 5.

2 System Model

Suppose there are multiple buyers (secondary network operators, SNOs) and sellers (primary network operators, PNOs) in a spectrum market. The SNOs are trying to acquire the idle spectrum from the PNOs in multiple trade windows. More specifically, the usage rights of the idle spectrum are to be given over to one of SNOs temporarily after wining of an auction, where multiple SNOs compete to acquire those spectrum resources through offline and online (in real-time) auctions. We denote $\mathcal{N} = \{1, 2, \ldots, N\}$ and $\mathcal{M} = \{1, 2, \ldots, M\}$ as the finite sets of PNOs and SNOs, respectively which operate in an area $\mathcal{R}$. Therefore, $\mathcal{N}$ represents the set of sellers and $\mathcal{M}$ the set of buyers in the region $\mathcal{R}$. For a cell in the region $\mathcal{R}$, $r \in \mathcal{R}$, total number of network operators $\equiv \mathcal{M} \cup \mathcal{N}$, without loss of generality. Denote $\mathcal{L}_i(t_h) = \{a_{i1}, a_{i2}, \ldots, a_{ij}, \ldots, a_{iL_i}\}$ be the set of auctions released from the ith PNO to be borrowed for a time duration t_h. For a time duration $[t_0, t_c] \equiv t_h$, the $M \in \mathcal{M}$ SNOs are actively seeking to borrow spectrum with demand $\mathcal{D}(t_h) = \{d_1, d_2, \ldots, d_M\}$ to increase the efficiency and quality of spectrum usage to their users. Let

$$\mathcal{A}(t_h) = \{a_{ij}(t_h), i = 1, 2, \ldots, N, j = 1, 2, \ldots, L_i\},$$

be a finite set of concurrent auctions from PNOs which are run by a spectrum regulator (SR) to be bidden by the SNOs. Let a_{ij} be the jth auction of ith PNO, which represents the amount of spectrum, latency and allowed transmit power and L_i be the number of auctions from the ith PNO. Now we make the following assumptions for modeling purpose:

1. one spectrum regulator (SR) running the auctions,
2. opening and closing time of auctions are concurrent and SNOs are allowed to make bids simultaneously,
3. multiple SNOs may have multiple choices of auctions to enter their bids,
4. amount, quality, transmit power and price of the auctioned spectrum blocks may vary from auction to auction,
5. PNOs and SNOs auction/bid strategies, requirements and utilization history are known to the third party spectrum regulator (SR),
6. an SNO after winning and opting out pays a penalty amount with a fair price set by the SR agreed by both the PNOs and SNOs before entering to the auctions, and

7. secondary users (borrower) obtain the network access credentials and access to the service through the PNO's (seller's) registration and authentication operations.

Based on the system and assumptions described above, SNOs first select the sets of closely matched feasible auctions according to the expected quantity, quality and transmit power according to their need. Then, SNOs find the optimal number of auctions to bid by minimizing the expected cost involved and budget at the pre-bidding stage. Once the optimal number of auctions is found, SNOs bid this optimal number of closely matched auctions. For a higher number of matched auctions than required, SNOs perform a utility-based ranking to select the best possible auctions. After bidding the selected optimal auctions, win and lose are performed via the Vickery auction mechanism. If an SNO wins a single auction then the SNO carries on with the auction and borrows the spectrum resources from the PNO. However, if there are multiple auctions won by an SNO for a region $r \in \mathcal{R}$ for a time duration t_h, then the SNO chooses the auctions with higher utility-based ranks generated previously of the won auctions (higher net utility surplus value) and opts out from the other auctions with penalty cost. Once bidding, win-lose and trading of auctions are settled, the spectrum regulator (SR) allocates the spectrum resources from the sellers (PNOs) to the buyers (SNOs).

3 Problem Formulation and Solution

3.1 Auction matching

In a dynamic spectrum market, secondary network operators (SNOs), buyers seek to borrow spectrum resources from primary network operators (PNOs) sellers by bidding suitable auctions. With a number of PNOs in the market, it is, therefore, a challenge for the SNOs to find out the most suitable auctions for their secondary users to bid for. The SNOs can reveal the closely matched auctions by comparing their demands with the amount of spectrum in the running auctions and also comparing the latency and transmit powers. This will lead the optimization problem to the participating SNOs for the bidding. To formulate the problem, we assume that there are $\sum_{i=1}^{N} L_i$ auctions, defined as,

$$\mathcal{A}(t_h) = \{a_{ij}(t_h), i = 1, 2, \dots, N, j = 1, 2, \dots, L_i\},$$

running and M SNOs are bidding some auctions for a particular duration of time $t_h = [t_0, t_c]$. Each SNO has a desired amount of spectrum to borrow for its users which is in the narrow range of $(\mathrm{d}_k, \mathrm{d}_k \pm \delta_k)$, $\delta_k \ll \mathrm{d}_k$, where d_k is the desired amount of spectrum for kth SNO and δ_k is the tolerance factor. The amount of latency and transmit power associated with the auction a_{ij} are $r_{ij}(t_h)$ and $l_{ij}(t_h)$.

However, not all auctions are suitable for an SNO for the duration of time t_h. Therefore, a bidder (SNO) will find only the closely matched auctions. Close matching, measured in distance, is defined as the difference in the amount of spectrum available and required taking into account the transmit power and latency. Formally, the matched auction selection problem can be formulated as

$$(\mathcal{P}_1): \quad \max \quad \sum_{i=1}^{N}\sum_{j=1}^{L}\sum_{t_h=1}^{T} a_{ij}(t_h)\, x_{ij}(t_h) \tag{1}$$

$$\text{s.t.} \qquad (|q_{ij}(t_h) - \mathrm{d}_k(t_h)|)\, x_{ij}(t_h) \le \delta_k(t_h)\; \forall\, i, j \tag{2}$$

$$l_{ij}(t_h)\, x_{ij}(t_h) \le \zeta_k(t_h)\; \forall\, i, j \tag{3}$$

$$r_{ij}\, x_{ij}(t_h) \le \gamma_k\; \forall\, i, j \tag{4}$$

$$\sum_{i=1}^{N}\sum_{j=1}^{L}\sum_{t_h=1}^{T} q_{ij}(t_h)\, x_{ij}(t_h) \le \mathrm{d}_k\; \forall\, k, \tag{5}$$

$$w\left(x_{i1}(t_h), x_{i2}(t_h), \ldots, x_{ij}(t_h)\right) \le \nu_k\; \forall\, i, j \tag{6}$$

$$\sum_{j=1}^{L}\sum_{k=1}^{M} u_{hk}\, x_{ij}(t_h) \ge \lambda_u,\; \forall k \in \{1, 2, \ldots, M\} \tag{7}$$

$$x_{ij}(t_h) \in \{0, 1\}\; \forall\, i, j. \tag{8}$$

where $q_{ij}(t_h)$ is the amount of spectrum (number of channels) in the jth auction of the ith PNO for the time period t_h, d_k is the amount of spectrum demanded by the kth SNO with minimum required latency $\zeta_k(t_h)$ and transmit power γ_k; $w(\cdot)$ is the highest bandwidth gap between selected carriers; ν_k is the maximum allowed threshold gap of an SNO; u_{hk} is the utilization history of the kth bidder (SNO); λ_u is the utilization history threshold; x_{ij} is the binary $\{0,\, 1\}$ decision variables whether to bid or not, l_{ij} and $r_{ij} \in \mathbb{C}^T$, $\forall i$, are latency and transmit power associated with the auction for the spectrum resources $a_{ij}(t_h)$. We denote the time duration of the corresponding auctions by $t_h \in T$.

Handling non-contiguity: Often SNOs require a higher amount of the spectrum resources than the amount of resources available in individual concurrent auctions for a time duration t_h. In that case, SNOs bid for multiple auctions simultaneously to acquire the required amount of resources and the optimization $(\mathcal{P}_1)$ turns into the combinatorial problem $(\mathcal{P}_1^*)$. This gives additional opportunities to the SNOs to bid for multiple auctions. However, it results in acquiring non-contiguous spectrum resources that obstruct seamless data transmission and achieve a lower throughput than expected with the same amount of resources. To include non-contiguous spectrum resources, we add two constraints (5) and (6) in $\mathcal{P}_1$.

Utilization efficiency: One of the objectives is to increase the efficiency of the unused and under-utilized spectrum resources of the operators (PNOs). To maximize the efficiency of the bandwidth, at the beginning of each auction round, we filter the total number of bidders according to the highest utilization coefficient. This is to restrict the auction to a smaller set of bidders which reduces contention for resources. We assume that each bidder has a utilization history, a parameter denoted by $\widehat{u}_{hk}$, defined as

$$\widehat{u}_{hk} = \exp\left(\frac{u_k}{u_k + \mathcal{N}}\right),$$

where u_k as the number of times the utilization is below a threshold λ_u and $\mathcal{N}$ is the length of previous successful bids of the bidder k. For example, if a bidder won two auctions successfully on two different occasions for time duration t_1 and t_2 respectively. Then, $X = \sum_{i=1}^{n} t_i = t_1 + t_2$. Therefore, it essentially implies the total duration of all previous successful bids. Furthermore, given these information, the spectrum regulator sorts bidders according to their utilization coefficient, and only allows a subset of the bidders to compete for the spectrum, denoted as B^*. The excluded bidders who do not meet the utilization threshold, are permitted to bid for the remaining, unsold bandwidths. Therefore, the utilization constraints can be formulated as shown in (7).

In the proposed auction trading mechanism, one of the overall goals is to increase spectral efficiency through higher utilization by secondary users from unused/under-utilized spectrum resources from the PNOs. Since regulation and process of auctions occur by the third party spectrum regulator (SR), PNOs receive a fair amount for leasing contracts (spectrum) set by the PNOs, which is regulated by the SR. However, if a PNO wants to maximize its revenue/profit rather than utilization of the resources by the SNOs, then it can also be done through the proposed mechanism by setting a very low utilization history threshold or a value '0'. This will allow all bidding SNOs equally probable to win a bid.

Solving $\mathcal{P}_1$: The problem $\mathcal{P}_1$ is a matching problem where the maximization is performed over all combinations ($\mathcal{N} \otimes \mathcal{M} \otimes_{i=1}^{N} L_i$). In this case, the optimizer finds the most suitable auctions for the SNOs satisfying the demand constraint (2), the latency constraint (3) and the power transmission constraint (4). The optimization problem ($\mathcal{P}_1$) is a matching problem, more specifically, a $0-1$ integer programming problem, which can be solved using the branch-and-bound algorithm that provides the global optimum. However, due to extra constraints (two contiguity) (5) and (6) and the utilization history constraint (7) in $\mathcal{P}_1$, the optimization problem becomes an NP-hard problem and computationally intractable to solve due to significantly larger search-space. Therefore, we propose a heuristic algorithm to solve the $\mathcal{P}_1$ optimization problem.

3.2 Price-based optimal number of auctions

Once SNOs (buyers) finds the set of closely matched auctions according to their need for their users, the problem is then how many of these auctions to bid for. Each auction has a specific reserve price i.e., a minimum asking price. The reserve cost allows the buyers (SNOs) to give the full details of auctions, for example, the total amount of spectrum resources, borrowing time duration, borrowing conditions, characteristics of the resources, etc. In a decentralized market, SNOs will not know the prices ahead for any given trade window and they must express their interest to be informed of the reserve price of spectrum resources to be bid. An SNO chooses the number of inquiries to make, however, each inquiry costs some amount. Once the minimum reserve prices are revealed after making an inquiry with some costs, the SNO finds more suitable auction sets based on the quoted lowest minimum reserve price. The problem is now to find the optimal number of inquiries n^* to make by an SNO which minimizes the total cost, where the total cost equals the expected borrowing cost plus an inquiry cost. Let us assume that the minimum reserve price p_{ij} for an inquiry to the jth auction of the ith PNO is a random variable that follows a distribution with a cumulative distribution function (CDF) $F(p_{ij}) = \mathbb{P}(P \leq p_{ij})$. Then,

$$\mathbb{P}(P > p_{ij}) = 1 - F(p_{ij}).$$

With n independent inquiries,

$$\mathbb{P}(P > p_{ij}) = [1 - F(p_{ij})]^n ,$$

is the probability of the minimum reserve price. Hence, the empirical cumulative distribution (ECDF) of the lowest minimum reserve price for n independent inquiries is

$$F_n(p_{ij}) = 1 - [1 - F(p_{ij})]^n.$$

Let c_{ij} be the reserve cost to bid a unit of the spectrum to the jth auction from the ith PNO. Now the expected total reserve cost of an SNO to borrow a unit of the spectrum is

$$\begin{aligned}
\mathbb{E}\big[c_{ij}(1,n)\big] &= \int_{\alpha_1}^{\alpha_2} p_{ij} \cdot \mathrm{d}F_n(p_{ij}),\\
&= \int_{\alpha_1}^{\alpha_2} p_{ij} \cdot f_n(p_{ij})\ \mathrm{d}p_{ij},\\
&= \int_{\alpha_1}^{\alpha_2} \left[1 - \mathbb{P}(P \leq p_{ij})\right]\ \mathrm{d}p_{ij}, = \int_{\alpha_1}^{\alpha_2} \left[1 - F(p_{ij})\right]^n\ \mathrm{d}p_{ij},
\end{aligned}$$

where α_1 and α_2 are the lowest and highest expected reserve price, respectively. Therefore, the expected total reserve cost to borrow $\mathrm{d}_k(t_h)$ amount of spectrum is

$$\mathbb{E}\big[c_{ij}(\mathrm{d}_k(t_h), n)\big] = \mathrm{d}_k(t_h) \cdot \int_{\alpha_1}^{\alpha_2} \left[1 - F(p_{ij})\right]^n\ \mathrm{d}p_{ij}. \tag{9}$$

We now proceed to formulate the optimization problem ($\mathcal{P}_2$) to find the optimal number of auctions to bid which can be mathematically expressed as,

$$\begin{aligned}
(\mathcal{P}_2): \quad & \min \mathbb{E}\left[c_{ij}(\mathrm{d}_k(t_h), n)\right]\\
&= \min \left[\mathrm{d}_k(t_h) \cdot \int_{\alpha_1}^{\alpha_2} \left[1 - F(p_{ij})\right]^n\ \mathrm{d}p_{ij}\right]
\end{aligned} \tag{10}$$

$$\begin{aligned}
\text{s.t.} \qquad c_{ij} \leq \ & b_0,\\
\alpha_1 \leq p_{ij} \leq \ & \alpha_2,
\end{aligned}$$

where b_0 is the total allocated expenditure for investigation (initial budget) of the kth SNO.

For a reasonable price dispersion ($\alpha_1 < \alpha_2$), solving the above optimization problem ($\mathcal{P}_2$) yields a large amount of savings and a good return of investment for the buyers (SNOs). A similar logic can be applied for the quantity $\mathrm{d}_k(t_h)$, as spectrum demand is high the SNOs find more profitable to solve the optimization problem ($\mathcal{P}_2$). To this end, an SNO seeks to obtain the optimal number of bids n^* so that the reserve cost is minimal and does not exceed the initial budget b_0. Mathematically, we can express as

$$\begin{aligned}
n^* &= \underset{n}{\operatorname{argmax}} \left(\min_k \mathbb{E}\left[c_{ij}(\mathrm{d}_k(t_h), n)\right] \leq b_0\right)\\
&= \underset{n}{\operatorname{argmax}} \Bigg(\min_k \left[\mathrm{d}_k(t_h) \cdot \int_{\alpha_1}^{\alpha_2} \left[1 - F(p_{ij})\right]^n\ \mathrm{d}p_{ij}\right] \leq \sum_i \sum_j c_{ij} a_{ij}\Bigg).
\end{aligned} \tag{11}$$

3.3 Utility-based ranking of auctions

Often buyers (SNOs) bid multiple auctions to ensure that they win at least one auction to provide the required or additional spectrum resources for their users to guarantee a certain level of service or to improve the quality of service. If an SNO finds the number of closely matched auctions higher than the optimal number to bid or wins more than one auction and plans to borrow through one auction, then the SNO needs to decide– through which auctions they should borrow resources. To choose the most economical and efficient auction to borrow resources, the SNO performs a utility-based analysis to ensure that they borrow resources according to their users need and consistent with their features. Consider an SNO $k \in \mathcal{M}$ with the features represented by two vectors π_k and ψ_k with $\pi_k, \psi_k \in \mathbb{R}_{\geq 0}$. The vector π_k describes any SNO features that affect the cost of service provision of an auction from a seller (PNO). The vector ψ_k represents all features of the SNOs relevant to its requirements and constraints, such as the number of channels, cost, etc.

Suppose p_{ij} is the price for the jth auction available from the ith PNO described by plan features ϕ_{ij}, where ϕ_{ij} includes taxes, penalties and so forth. The uncertainty of the future characteristics (e.g. traffic, leasing agreement) and strategies (e.g. security, regulation) of the kth SNO is defined by $s_k \in \mathbb{R}_{\geq 0}$, where the distribution of uncertainty over the strategy of the (ij)th auction is $G(s_k|\, y_{ij}, p_{ij}, \phi_{ij})$. The expected utility of using y_{ij} amount of spectrum in auctions $a_{ij} \in (y_{ij}, p_{ij}, \phi_{ij})$ by a SNO (π_k, ψ_k) is

$$\begin{aligned}\nu(y_{ij}, p_{ij}, \phi_{ij}) &= \int U(y_{ij}, p_{ij}, \phi_{ij}, s_k, \pi_k, \psi_k)\, \mathrm{d}s_k\, \mathrm{d}\pi_k\, \mathrm{d}\psi_k \\ &= \int U(y_{ij}, p_{ij}, \phi_{ij}, s_k) \cdot H(\pi_k, \psi_k \,|\, y_{ij}, p_{ij}, \phi_{ij}, s_k)\, \mathrm{d}s_k\, \mathrm{d}\pi_k\, \mathrm{d}\psi_k, \end{aligned} \tag{12}$$

where $U(\cdot)$ and $H(\cdot)$ are the multivariable utility functions satisfying the independence, completeness, continuity and transitivity properties of a utility function [8]. The equation (12) can further be simplified by separating the uncertainty s_k for given price, amounts to borrow and plan features as

$$\nu(y_{ij}, p_{ij}, \phi_{ij}) = \int U(y_{ij}, p_{ij}, \phi_{ij}) \cdot G(s_k|\, y_{ij}, p_{ij}, \phi_{ij}) \cdot H(\pi_k, \psi_k \,|\, y_{ij}, p_{ij}, \phi_{ij}, s_k)\, \mathrm{d}s_k\, \mathrm{d}\pi_k\, \mathrm{d}\psi_k. \tag{13}$$

where $H(\cdot)$ is the joint function of the vectors π_k and ψ_k.

If the two vectors π_k and ψ_k remain constant over the strategies of the PNOs, then (13) becomes

$$\nu(y_{ij}, p_{ij}, \phi_{ij}) = K \int U(y_{ij}, p_{ij}, \phi_{ij}, s_k) \cdot G(s_k|\, p_{ij}, \phi_{ij})\, \mathrm{d}s_k. \tag{14}$$

where K is the constant. However, the SNO will choose the auction a_{ij} under the strategy with given π_k and ψ_k, which gives

$$\begin{aligned}\nu(y_{ij}, p_{ij}, \phi_{ij}|\, \pi_k, \psi_k) &= \int U(y_{ij}, p_{ij}, \phi_{ij}, s_k|\, \pi_k, \psi_k)\, \mathrm{d}s_k \\ &= \int U(y_{ij}, p_{ij}, \phi_{ij}|\, \pi_k, \psi_k) \cdot G(s_k|\, p_{ij}, \phi_{ij})\, \mathrm{d}s_k. \end{aligned} \tag{15}$$

For a set of given features (π_k, ψ_k) of the kth SNO, the utility obtained for borrowing y_{ij} amount of spectrum can be given by

$$U(y_{ij}, p_{ij}, \phi_{ij}) = 1 - \exp\left[-\left(\frac{y_{ij}}{\phi_{ij}}\right)^{p_{ij}}\right], \quad y_{ij} > 0. \tag{16}$$

The utility function (16) also satisfies all the common properties– independence, completeness, continuity and transitivity of a utility function [8]. The uncertainty function for a fixed amount of spectrum borrow y_{ij} with mean μ_{s_k} and standard deviation σ_{s_k} can have the following form

$$G(s_k \,|\, p_{ij}, \phi_{ij}) = \frac{1}{\sqrt{2\pi}\sigma_{s_k}} \exp\left[-\left(\tfrac{s_k - \mu_{s_k}}{\sigma_{s_k}}\right)^2\right], \quad -\infty < s_k < +\infty. \tag{17}$$

With the expected utility as defined in equation (15), an SNO chooses to bid in an auction that generates the highest expected utility

$$\nu(y_{ij}, p_{ij}, \phi_{ij}|\pi_k, \psi_k) \geq \nu(y_{lm}, p_{lm}, \phi_{lm}|\, \pi_k, \psi_k), \quad \forall l \in N, m \in L_N. \tag{18}$$

Suppose $c(y_{ij}, \phi_{ij})$ is the welfare loss for being not able to use the y_{ij} amount of spectrum by the kth SNO through the jth auction after wining under the plan feature ϕ_{ij} and is defined as

$$c(y_{ij}, \phi_{ij}) = \int L(y_{ij}, p_{ij}, \phi_{ij}, s_k) \cdot G(s_k | \, p_{ij}, \phi_{ij}) \, \mathrm{d}s_k, \tag{19}$$

where $L(\cdot)$ is the uncertainty-specific welfare loss of the ith PNO providing service through the jth auction to the kth SNO. The welfare loss function $L(y_{ij}, p_{ij}, \phi_{ij}, s_k)$ can take many forms: linear, quadratic, cubic, etc. The expected cost of the PNO depends on the SNO that determines the distribution $G(s_k | \, p_{ij}, \phi_{ij})$. We now define the net utility surplus (NUS) as

$$\mathcal{W} = \sum_{k \in \mathcal{M}} \sum_{ij \in \mathcal{A}''} I(w_{ijk}) \cdot \big[\nu(y_{ij}, p_{ij}, \, \phi_{ij} \, | \, \pi_k, \psi_k) - c(y_{ij}, \phi_{ij})\big], \tag{20}$$

where $I(w_{ijk})$ is an indicator function for the set of auctions won selected by the kth SNO. The quantity $\mathcal{W}$ is the standard unconstrained efficiency condition for each auction, which is used to order and choose auctions according to their higher NUS values.

3.4 Auction bidding and spectrum allocation

We suppose that there are M potential buyers (SNOs) bidding for the spectrum resources for each auction. There is a privately known value for the auctioned spectrum resources and each potential buyer (SNO) finds this private value as a random variable V, which represents an independent draw from the cumulative distribution function (CDF) $F_k(v)$ that is continuous and differentiable and also has a probability density function (PDF) $f_k(v)$ on a compact support $[v_{\min}, v_{\max}]$. To reduce clutter, let Z denote $V_{(1:M-1)}$, the highest valuation of $(M-1)$ bids from the $F_k(v)$; in symbols, $Z = \max\{V_1, V_2, \ldots, V_{(M-1)}\}$. The random variable Z represents the highest of a bidder's $(M-1)$ opponents at the auction. Given that valuations are distributed independently and identically, the CDF and PDF of Z are $F_Z(z) = F_k(z)^{M-1}$ and $f_Z(z) = (M-1)F_V(z)^{M-2} f_k(z)$, respectively.

The valuation of the kth SNO of the radio resources is independently and identically distributed according to the CDF $F_k(\cdot)$ on the interval $[\mathrm{d}_k(t_h) \, p_{ij}, \varepsilon_k]$. According to the proposed auction mechanism, each SNO submits a bid with price

$$Y_k \in \, [\mathrm{d}_k(t_h) \, p_{ij}, \varepsilon_k), \; \mathrm{d}_k(t_h) \, p_{ij} > 0.$$

All bids $[\mathrm{d}_k(t_h) \, p_{ij}, \varepsilon_k)$ are highly competitive. An SNO submitting the highest bid wins the auction and makes a payment equal to the second highest bid $Y_k^{(b-1)}$ or the reserved price $\mathrm{d}_k(t_h) \, p_{ij}$ in the case, where there is only one submitted bid. In the case, where two or more SNOs submit the same bid and it turns out to be the highest, then the tie is resolved arbitrary with a uniform randomization and the winner pay the highest bid $Y_k^{(b)}$. Any bid $Y_k < \mathrm{d}_k(t_h) \, p_{ij}$ is a noncompetitive bid, equivalent to not participating in the auction. Most importantly, an SNO bidding $Y_k < \mathrm{d}_k(t_h) \, p_{ij}$ receives a payoff of zero, irrespective of the others' bids (and even if it is the only bid). The payoff of an SNO, who values the radio resource as V_k and wins the auction with bid Y_k is $V_k - Y_k^{(b-1)}$. In this case, the unique Bayesian-Nash equilibrium [53], where all bidders adopt the bidding strategy is

$$\beta_{V_k} = \begin{cases} Y_k, & \text{if } V_k < \mathrm{d}_k(t_h)\, p_{ij}, \\ V_k - \displaystyle\int_{\mathrm{d}_k(t_h)\, p_{ij}}^{V_k} \left[\frac{F(z)}{F(V_k)}\right]^{[M-1]} dz, & \text{if } V_k \geq \mathrm{d}_k(t_h)\, p_{ij}, \end{cases} \tag{21}$$

where $F(V_k)$ is the CDF of the valuation of the kth SNO. The assumption that the SNOs may place multiple bids in multiple auctions to increase their chances of gaining spectrum access raises the need to consider aborted bids and hence the second case of equation (21) can be written as

$$\beta_{V_k} = V_k - \int_{\mathrm{d}_k(t_h)\, p_{ij}}^{V_k} \left[\frac{F(z)}{F(V_k)}\right]^{[M-M'-1]} dz, \quad \text{if } V_k \geq \mathrm{d}_k(t_h)\, p_{ij}, \tag{22}$$

where M' is the number of aborted SNOs from the auction a_{ij}.

There are penalties involved for opting out after winning an auction. The winner of more than one auction if decides to abort for some won auctions, pays an initial fixed penalty in form of monetary value plus additional amount based on the volume of resources in the bid for each aborted auction. The penalty function can take either a linear or an exponential form and written mathematically as,

$$Y_p = \begin{cases} c_0 + \displaystyle\sum_{i=1}^{p-1} c_i x_i, & \text{if linear}, \\ c_0 \exp\left(\displaystyle\sum_{i=1}^{p-1} c_i x_i\right), & \text{if exponential}, \end{cases} \tag{23}$$

where p $(\leq n)$ is the number of bids won, c_0 is the fixed penalty, c_i is the variable penalty per unit and x_i is the amount of bid resources. Once an auction is complete, the winner is allowed to use the spectrum for the specified time and within the spatial regions. This approach simplifies the mechanisms within the spectrum market, allowing the exchange of radio resources between operators (or between users and operators with some adjustments) and facilitating transactions. Algorithm 1 describes the full mechanism of the auction bidding, win, lose and management of auctions.

Overview of Algorithm 1 — In Algorithm 1 we provide the full mechanism of the auction bidding, including matching bidders' resource requirements with auctions, win/lose and final allocation. The matching adversaries' demand with auctions is determined in this step according to $\mathcal{P}_1$. As explained in $\mathcal{P}_1^*$, the non-contiguous aggregation is considered given the respective allowable gap between channels. In the first round of the auction, the priority is given to the bidders (B^*) with higher utilization history score, $\hat{u}_h$. In the case where there are unsold resources, following the first round of auctions, other adversaries (B_{res}) can participate in the second round, irrespective of their utilization history. In the second stage of the algorithm, SR finds the winner using a Vickrey type pricing mechanism and offers the winning auctions to the SNOs. However, an SNO may opts out after winning by paying a penalty cost set by the SR.

3.5 Computational complexity

In this section, the computational complexity of Algorithm 1 is described. The first block of Algorithm 1 uses a simple `for` loop with a binary classification of auctions to index them into a vector. Therefore, the

Algorithm 1: Spectrum management

1 Initialization: $\mathcal{U} = \{\widehat{u}_h^1, \widehat{u}_h^2, \ldots, \widehat{u}_h^M\}$; λ_u = utilization threshold; $B = \{b_1, b_2, \ldots, b_M\}$ (all bidders); $B^* = \emptyset$ (filtered list of bidders); $B_{\text{res}} = \emptyset$ (list of reserved bidders)
2 **for** $i = 1 : M$ **do**
3   **if** $\widehat{u}_h^i \geq \lambda_u$ **then**
4         $B^* \leftarrow B_i$
5   **else**
6         $B_{\text{res}} \leftarrow B_i$
7 Let $\mathbf{a} = \{\mathbf{a}_1, \mathbf{a}_2, \ldots, \mathbf{a}_N\}$ set of auctions for N PNOs; Number of bidders who meet the utilization threshold $1, 2, \ldots, |B^*|$; Number of bidders who do not meet the utilization threshold $1, 2, \ldots, |B_{res}|$; Set of auctions $1, 2, \ldots L_i$; Starting and closing auction time $[t_0, t_c]$; Minimum price matrix $\mathbf{P}_l$, $Y_k^{(b)} \leftarrow$ best price; $\mathbf{P}_l$, $Y_k^{(b-1)} \leftarrow$ second best price; $Y_k \leftarrow$ value of the jth bid
8 **for** $k = 1 : |B^*|$ **do**
9   **for** $i = 1 : N$ **do**
10   **for** $j = 1 : L_i$ **do**
11   Perform optimization: $\mathcal{P}_1$ %Auction matching
12   Index$[i, j, k] \leftarrow$ identity$(\mathbf{a})$
13 **while** $t_c \neq 0$ **do**
14   Index $\leftarrow$ matrix$[N, L_i]$ %Track winner (bidder-SNO)
15   Counter $\leftarrow$ matrix$[N, L_i]$ %Track total winner-SNO
16   **for** $k = 1 : |B^*|$ **do**
17   **for** $i = 1 : N$ **do**
18   **for** $j = 1 : L_i$ **do**
19                 $\mathbf{Y} \leftarrow$ Vector of all bids received $P_i \geq P_m$
20                 $Y_k^{(b)} \leftarrow$ best price for i, jth bid
21                 $Y_k^{(b-1)} \leftarrow$ second best price for (i, j)th bid
22   Index$[i, j, k]$ = identity$(\mathbf{a})$
23   Counter$[i, j, k]$ = sum(index)
24   **if** Counter$[i, j, k] = 0$ **then**
25   Do not Proceed
26   **else if** Counter$[i, j, k] = 1$ **then**
27   Proceed
28   **else**
29   Proceed with max utility surplus W
30   %Performing utility-based ranking Abort with penalty cost Y_p
31   Refund to Auction Winner = $Y_k^{(b)}$ - $Y_k^{(b-1)}$.
32   % Calculating the refund to Auction winner.
33   **if** $k \neq$ Winner **then**
34   kth SNO receives the bidding price.
35   **if** $|\mathbf{a}| \geq \mathbf{1}$ **then**
36   **for** $k = 1 : |B_{res}|$ **do**
37   Repeat from step 11 to 28
38 **return**

complexity of the loop is $\mathcal{O}(N \log n)$ as the loop variables are divided by a constant amount of times. However, the second and third blocks of Algorithm 1 are more complex. These take the selected auctions, say (N^*), as input for the analysis. We see that the analysis is performed in two stages. In the first stage, optimization $\mathcal{P}_1$ is performed to select the best-matched auctions. If $\mathcal{P}_1$ is considered, which is a binary optimization problem then the complexity is $\mathcal{O}(N^* \log n)$. Instead of $\mathcal{P}_1$, if $\mathcal{P}_1^*$ is performed then we have to perform optimization over all possible combinations to select those having constraints (5) satisfied. In this case, the complexity of the first part of the algorithm with the number of bidders B^* is

$$B^* \cdot \sum_{i=1}^{N^*} C_i^{N^*} = B^* \left[\left(\sum_{i=0}^{N^*} C_i^{N^*} \right) - 1 \right] = B^* \left(2^{N^*} - 1 \right).$$

The second part of the algorithm performs the win-lose of the auctions based on Vickery algorithm and allocation of spectrum resources, in which the two inner `for` loop with the `if else` has a complexity of $\mathcal{O}(N^* \log n)$. Therefore, the complexity of the second part becomes $\mathcal{O}(B^* N^* \log n)$ and the total complexity of Algorithm 1 is

$$B^* \left(2^{N^*} - 1 \right) + B^* N^* \log n = B^* \cdot 2^{N^*}.$$

Hence, the computational complexity of Algorithm 1 is $\mathcal{O}(B^* \cdot 2^{N^*})$.

4 Numerical results

To validate and verify our proposed spectrum management framework, we have performed numerical investigations. In this section, we shall discuss the performances of the framework which is designed to select and manage auctions for spectrum allocation under various parameter settings. Our proposed framework allows PNOs to release all of their unused resources for bidding by SNOs. We have first discussed how SNOs select for the most suitable set of auctions considering the amount and quality required under two different scenarios: contiguous and non-contiguous carrier aggregation. Once the most suitable auctions are selected then we have demonstrated how to find the optimal number of auctions to bid given that an SNO has a fixed amount of budget to explore auctions to bid. Then, we have shown how the spectrum regulator allocates the channels to the SNO through fair pricing after bidding the suitable auctions by an SNO. Finally, we have investigated how an SNO can choose the best set(s) of auctions from different PNOs based on net utility surplus if it wins more resources than it requires.

4.1 Best set of auctions and contiguous & non-contiguous consideration

One of the primary goals of the framework is to find a set of suitable auctions for the adversaries according to their spectrum demand, expected latency and transmit power as presented in $\mathcal{P}_1$ and $\mathcal{P}_1^*$. The optimization problem $\mathcal{P}_1$ selects the set of auctions when the demand for an SNO falls with the threshold value $\delta_k(t_h)$. This optimization problem finds the solution in polynomial time. The channel aggregation in modern cellular technologies adds another dimension to the problem. Without carrier aggregation, the choices for bidders is limited to only the resource blocks which match the demand, creating more contentions per resource block. To tackle this problem, we proposed and solved the optimization problem $\mathcal{P}_1^*$. The proposed optimization problem is a combinatorial optimization problem and is solved using a non-polynomial time algorithm. Here, bidders, are allowed to bid for various sizes of resource blocks, to meet their demands. Using contiguous and non-contiguous aggregation is largely dependent on the user equipment capability. Figure 1 clearly shows that the available space of auctions increases when we consider non-contiguous channel aggregation compared to its contiguous counterpart. This is more notable when the demand is greater as shown in Figure (1—right), where the demand is 200MHz. Moreover, the gap between carriers considered for non-contiguous channel bonding plays an important role, where the selection space is higher (800MHz), as can be seen in Figure 2. During simulation, we have also considered a range of utilization history of the SNOs (bidders) and tested several scenarios by taking different utilization history threshold values.

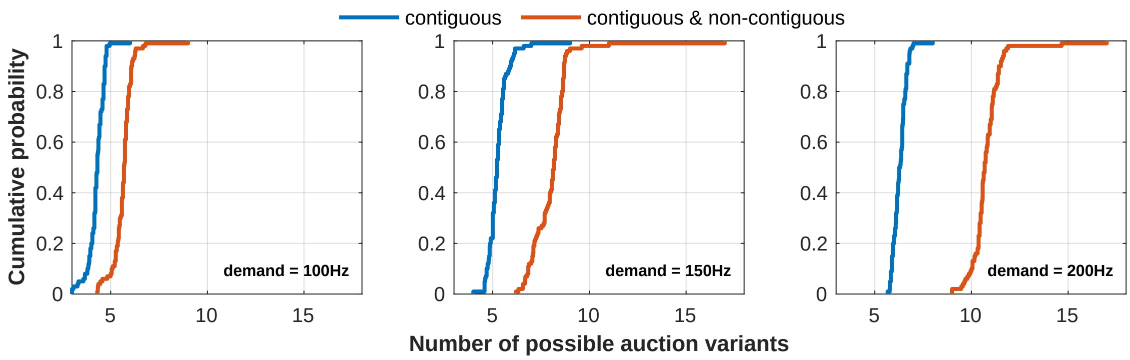

Fig. 1: Finding the number of auctions using contiguous and non-contiguous carriers aggregation with various demands.

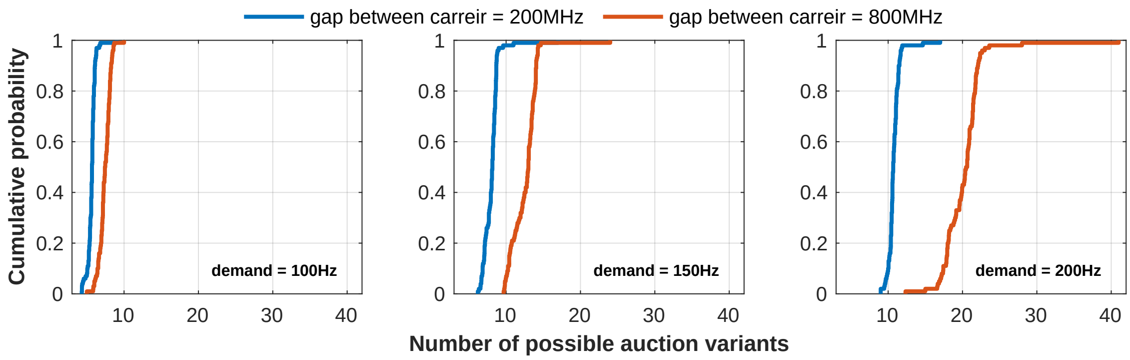

Fig. 2: Finding the number of auctions using non-contiguous carriers aggregation with various demands and gaps between carriers.

4.2 Minimizing the reserve cost through bidding for a larger number of auctions

We have assumed that concealed auctions involve a preset reserve cost, which allows the bidder to access more details of items being auctioned (e.g., time duration, borrowing conditions, etc.) if the cost is paid. Once a suitable set of auctions is obtained, an SNO aims to bid for an optimal number of auctions, minimizing costs. With a higher number of bids, more options are available to the bidders and it allows them to bid for the cheaper auctions. Consequently, the average reserve cost decreases with the higher number of bids. On the other hand, a higher number of bids would naturally incur higher bid request costs. An optimal solution is, therefore, required, which would allow the bidders to choose the optimal number of bids with a fixed budget to reveal actual bidding costs with other necessary information about the auctions. As expected, Figure 3 shows that the reserved cost decreases with the higher number of auctions for normal price distribution, under various demands.

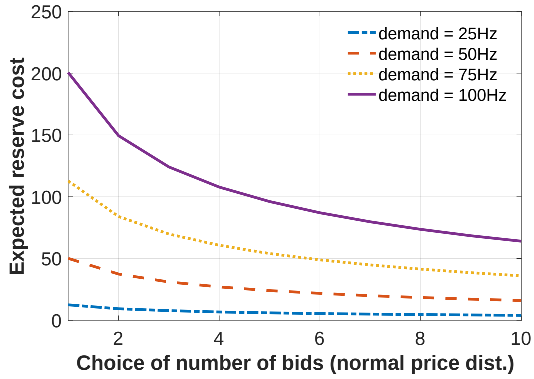

Fig. 3: Expected reserved cost for a different choice of bids for normal distribution

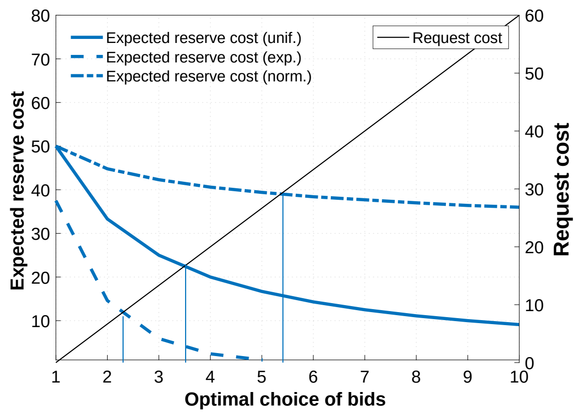

Fig. 4: Optimal choice of number of bids for different distribution and limited budget

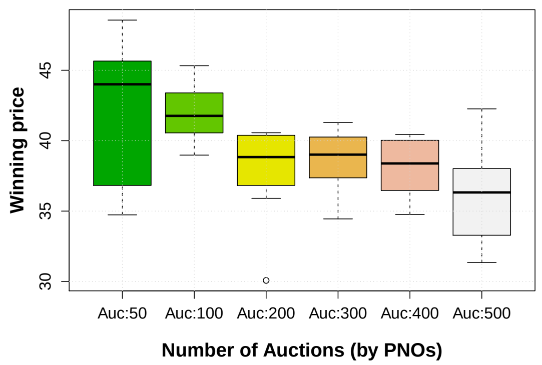

Fig. 5: Auction winning price against number of concurrent auctions

We also investigate the impact of contention between SNOs on the winning cost. From Figure 6, we can see that the winning price increases with the number of SNOs, however, we observe that due to the cap on maximum cost set to 100 units, the winning price is normalizing as it reaches the maximum value.

4.3 Bidding for the optimal number of auctions

The sensitivity of the expected costs is verified for different distributions of reserve prices with varying demand. We have performed experimentation with uniform, exponential, normal price distribution of auctions. We observed that the expected cost decreased with the number of auctions available. With historic bidding data,

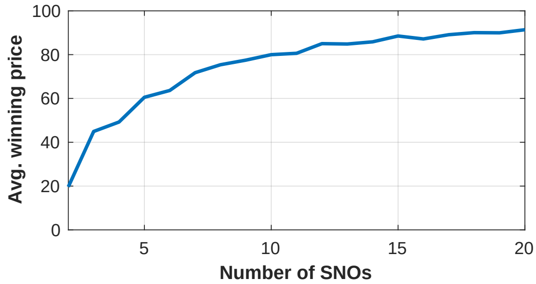

Fig. 6: Number of bidding SNOs vs the average winning price per SNO. The cost per auction is set between 10 and 100 units.

the bidder would be able to use empirical distribution, otherwise, the uniform distribution would be a potential candidate of the price distribution. A bidder (buyer) at this stage would look for an optimal number of auctions to bid with a limited budget which is addressed in $\mathcal{P}_2$ with the solution being given by the equation (11). With a limited auction searching budget, buyers (SNOs) can find the optimal number of auctions to bid. Figure 4 show that the buyers can find the optimal number of auctions to bid with their limited budget.

To study the impact of the number of available auctions in the market on the winning price, we investigate the auction which is presented in Algorithm 2. We varied the number of available auctions to be between 50 to 500. As shown in Figure 5, the average winning prices decrease with the higher number of auctions available in the market.

4.4 Choosing the auctions with maximum NUS

Intuitively, the higher amount spent in auction bidding increases the winning of the higher amount of spectrum resources that are to be used by the secondary users and increases the net utility surplus (NUT). Figure 7 shows an example of the ranking of five auctions won by a bidder. In that case, the bidder drops out from auctions with penalty cost and select the one (auction # 4) with the highest net utility surplus value.

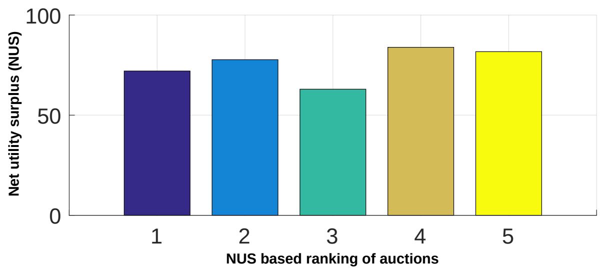

Fig. 7: Ranking of five auctions based on net utility surplus

5 Conclusion

In this research, a spectrum management framework for the secondary use of under-utilized spectrum resources from the sellers was proposed. It allows multiple sellers to run concurrent auctions to be bid by multiple buyers through a spectrum regulator. We examined the effect of varying demand of the buyers in terms of auction amount, spectrum bandwidth contiguity, price of auctions and buyers budget to bid optimal auctions, which have not been investigated previously. We investigated the best choice of selected auctions for a buyer who maximizes the net utility surplus in case of multiple wins by the buyers. The proposed scheme finds the optimal number and the set of auctions for secondary network operators (buyers) by minimizing enquiry cost within a budget by performing a search on a set of concurrent auctions from primary operators (sellers) while maintaining the qualify and efficiency of the spectrum resources. Having set fair prices by the spectrum regulators consistent with sellers' policy and auction managed by a Vickery type mechanism, the required secondary spectrum can be achieved by secondary network operators (buyers) if won an auction or multiple auctions. Our results show that the secondary spectrum can be managed fairly and flexibly by spectrum regulators if network operators (both primary operators who are sellers and secondary operators buyers) consider increasing spectrum efficiency while maintaining other effects such as fair competition and subscribers' surplus. The net utility surplus (NUS) would be used for choosing not only the best auctions but also the best possible sellers under non-auction based trading of resources (not necessarily telecommunication resources), sharing and leasing resources, etc. Although the study emphasized the secondary service providers' (buyers') interest to manage and increasing the efficiency of the secondary spectrum it indicates the overall increase of spectral efficiency.

Declarations

Funding:

There is no funding source to declare for this study.

Conflicts of interest:

We declare that we have no conflict of interest.

Availability of data and material:

There is no data involved in this study. Results are based on simulated data.

Code availability:

The codes will be available upon request from the corresponding author.

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