

# An Enhanced Smith-Predictor-based Control System for delayed MIMO Processes and its Use on a CSTR with Multiple Time-delays

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**Abstract**—This paper aims to enhance Smith-predictor-based control systems (SPCSs) for multi-input multi-output (MIMO) time delay processes. Conventional SPCSs for MIMO processes have an array of classical feedback controller(s). In practice, these controllers receive error signal(s), calculated with deducting a predicted output by a Smith predictor from a reference signal at the time of operation. Investigations on underperformance of conventional SPCSs identified two major shortcomings: (i) design of classical feedback controllers is based on trade-off, and their use may lead to winding phenomenon, these adversely influence SPCSs performance, (ii) a predicted output by a Smith predictor belongs to a time in the future and does not concurrent with the reference at the time of operation. That is, in conventional SPCSs, the control error is generated with use of two asynchronous signals. This paper proposes an enhanced SPCS design method for MIMO time-delay systems based on two enhancements to tackle the aforementioned dual shortcomings. The proposed control system evidently outperforms a conventional SPCS with internal model control (IMC) proportional-integral-derivative (PID) feedback controllers. The case study is a catalytic stirred tank reactor (CSTR) with three inputs (feed and water flow rates and auxiliary temperature), two outputs (output flow concentration and temperature) and three time delays. The presented model of the CSTR is more comprehensive than any CSTR model found in the literature.

**Keywords**—Smith Predictor, Time Delay System, CSTR, MIMO, Feedforward, Asynchrony

## I. INTRODUCTION

Many dynamic systems witness dead-time or delay and are categorised as time-delay systems [1, 2]. These include chemical processes, engines, manufacturing and telecommunication systems, etc. [3, 4]. Time-delay is known as a main source of instability and poor performance in control systems [5]. Prior to tackle this control issue, two types of time-delay should be differentiated: Type 1, where states of a system influence their time derivative with a delay. The source of this type of delay is the time needed to exchange data between devices in electrical/communication networks. Lyapunov-Krasovskii and Razumikhin theorems can be employed to deal with this type of delays [1]. Type 2 delay concerns the time needed for the control inputs to influence the time derivative of the system states [2]. This type of time-delay is a prevalent issue in process control [6], and is the focus of this paper.

Two main approaches have been employed to deal with Type 2 time-delays: Pade approximation and predictive methods. There is a third approach only usable to deal with

very short Type 2 delays (e.g. <10 ms), e.g. in electrical vehicles and drones [7], which is outside the scope of this paper. In the first approach, the term presenting the delay within the model (i.e. an exponential term in Laplace domain) is replaced by its approximate transfer function (an all pass filter in Laplace domain) using Pade technique [6]. Then, the resultant model is used for control system design. Pade approximation leads to model inaccuracy and transforms a stable minimum-phase system to a non-minimum-phase one with a higher order [8]. While, predictive methods, such as Smith predictors [9], do not have these drawbacks. As a result, they are expected to exhibit higher performance.

SPCSs, a well-established category of predictive control, have been developed for stable/unstable and SISO/MIMO systems [5, 10, 11] and systems with long delays [12]. As a result, Smith predictors were suggested to be a part of control solution for any stable or unstable time-delay system [2, 13].

Despite all the advantages of SPCSs, there are reports in the literature about the superiority of feedback control systems based on Pade approximation over SPCSs. For example, in control of linear systems with identifiable time-varying delays [14]. The causes of such unexpected underperformance of SPCSs were investigated in for single-input single output (SISO) systems in [10]. This research extends this investigation to MIMO systems, identifies inherited shortcomings of currently prevalent design approach and propose enhancements to address the identified shortcomings. The case study of this research is full-scale tracking control of a CSTR with three inputs, two outputs and three time delays. The employed CSTR model is unprecedentedly inclusive of system details.

## II. MIMO SOLUTION

This section aims to develop enhanced SPCS for linear MIMO time-delay processes:

$$\begin{bmatrix} y_1(k) \\ \vdots \\ y_{n_y}(k) \end{bmatrix} = \begin{bmatrix} G_{11}(z)z^{-r_{11}} & \dots & G_{1n_u}(z)z^{-r_{1n_u}} \\ \vdots & \ddots & \vdots \\ G_{n_y1}(z)z^{-r_{n_y1}} & \dots & G_{n_y n_u}(z)z^{-r_{n_y n_u}} \end{bmatrix} \begin{bmatrix} u_1(k) \\ \vdots \\ u_{n_u}(k) \end{bmatrix}, \quad (1)$$

where  $n_u$  and  $n_y$  are number of inputs and outputs respectively, and  $r$  stand for delay order.

Ogunnaike and Ray extended original SPCS to MIMO linear systems[15]. Their works leads to a control law of (2) for (1):

$$u_j(k) = \sum_{i=1}^{n_y} C_{ij}(z) (y_{di}(k) - y_{pi}), \quad (2)$$

where all  $C_{ij}$  controllers are classical controllers, and (3) and (4) results in the predicted output components,  $y_{pi}$  s:

$$y_{pi} = \sum_{j=1}^{n_u} \hat{G}_{ij}(z) u_j(k) (1 - z^{-r_{ij}}) + y_i(k), \quad (3)$$

$$\begin{bmatrix} y_{p1} \\ \vdots \\ y_{pm_y} \end{bmatrix} = \overbrace{\begin{bmatrix} \hat{G}_{11}(z)(1-z^{-r_{11}}) & \dots & \hat{G}_{1n_u}(z)(1-z^{-r_{1n_u}}) \\ \vdots & \ddots & \vdots \\ \hat{G}_{n_y,1}(z)(1-z^{-r_{n_y,1}}) & \dots & \hat{G}_{n_y,n_u}(z)(1-z^{-r_{n_y,n_u}}) \end{bmatrix}}^{\text{predictor matrix}} \times \begin{bmatrix} u_1(k) \\ \vdots \\ u_{n_u}(k) \end{bmatrix} + \begin{bmatrix} y_1(k) \\ \vdots \\ y_{n_y}(k) \end{bmatrix}, \quad (4)$$

Use of an array of classical controllers, alike (2), is still the dominant approach for SPCS of MIMO systems[16]. For the system of (1) and the control law of (2), [15] showed that delays do not influence the closed loop stability, if discrepancy of  $G_{ij}(z)$  and  $\hat{G}_{ij}(z)$  is negligible for all values of  $i$  and  $j$ . In other words, if  $C_{ij}$  controllers can stabilise the delay-less system, they can stabilise (1) too.

The control law of (2), as a widely accepted form of an MIMO SPCS, has the following shortcomings:

- (i) Classical controllers of  $C_{ij}$  suffer from trade-off and/or windup. Design of classical controllers is a trade-off between performance and the steady-state error. Moreover, windup phenomenon (the influence of actuator's saturation on integrals [17]) happens to classical controllers. Both trade-off-based design and windup (as well as known anti-windup components) may negatively affect control response and push it away an optimal behaviour.
- (ii)  $y_{pi}$  and  $y_{di}(k)$  are asynchronous in (7). In order to illustrate asynchrony, let us extend (3) to (5),

$$y_{pi} = \sum_{j=1}^{n_u} \hat{G}_{ij}(z) u_j(k) (1 - z^{-r_{ij}}) + \overbrace{\sum_{j=1}^{n_u} G_{ij}(z) u_j(k) z^{-r_{ij}}}^{y_i(k)} \stackrel{G_{ij} \approx \hat{G}_{ij}}{\Rightarrow} y_{pi} \approx \sum_{j=1}^{n_u} \overbrace{G_{ij}(z) u_j(k)}^{y_{pij}} = \sum_{j=1}^{n_u} y_{pij} \text{ or } y_{pij} \approx G_{ij}(z) u_j(k), \quad (5)$$

where  $y_{pij}$  is a constituent of  $y_{pi}$  influenced by  $u_j$ . In addition, from (1),

$$y_i(k) = \sum_{j=1}^{n_u} \overbrace{G_{ij}(z) u_j(k) z^{-r_{ij}}}^{y_{ij}(k)} = \sum_{j=1}^{n_u} y_{ij}(k) \Rightarrow y_{ij}(k+r_{ij}) = G_{ij}(z) u_j(k), \quad (6)$$

where  $y_{ij}$  is the constituent of  $y_i$  influenced by  $u_j$ .

Comparison of (5) and (6) shows that, in conventional MIMO SPCS, demonstrated by (2) and (3), the constituents of predicted output ( $y_{pij}$  s) do not belong to the same time as the constituents of output,  $y_{ij}(k)$ . Indeed, they are synchronous with  $y_{ij}(k+r_{ij})$ . As a result, in conventional MIMO SPCS, two

asynchronous variables ( $y_{di}$  and  $y_{pi}$ ) are compared to find the control error, which is used to produce the control command. Timewise, considering  $t_s$  as the sample time,  $y_{di}$  and  $y_{pi}$  are  $r_{ij} \times t_s$  seconds apart.

In order to address the aforementioned dual shortcomings, two enhancements are proposed for MIMO systems. Enhancement 1 removes the need for classical feedback controllers (shortcoming 1), and Enhancement 2 tackles asynchrony (shortcoming 2).

**Enhancement 1 MIMO:** A feedback-feedforward control law of (7) replaces the feedback controller of  $C_{ij}$  in (2).

$$u_j = u_j^* + \sum_{i=1}^{n_u} K_{ij} (x_{di} - x_{pi}), j = 1 : n_u, \quad (7)$$

where  $K_{ij}$  is an element of  $\mathbf{K}$  found through pole placement.  $x_{di}$  and  $x_{pi}$  are the states calculated based on  $y_{di}$  s and  $y_{pi}$  s.  $u_j^*$  is an input at the desired status. The following paragraph justifies (7).

As to [15], when Smith predictors are used, providing that the discrepancy of  $G_{ij}$  and  $\hat{G}_{ij}$  is negligible, a control system which stabilises the delay-free process ((1) when  $\forall r_{ij}=0$ ) will stabilise the original time-delay process, (1) with any values for  $r_{ij}$ . Let us assume (8) is a continuous state space model of the delay-free system:

$$\begin{cases} \dot{\mathbf{X}} = \mathbf{A}\mathbf{X} + \mathbf{B}\mathbf{U}, \\ \mathbf{Y} = \mathbf{C}\mathbf{X}. \end{cases} \quad (8)$$

At the desired status,

$$\begin{cases} \dot{\mathbf{X}}_d = \mathbf{A}\mathbf{X}_d + \mathbf{B}\mathbf{U}^*, \\ \mathbf{Y}_d = \mathbf{C}\mathbf{X}_d. \end{cases} \quad (8)$$

By subtracting (8) from (9):

$$\begin{cases} \dot{\mathbf{X}}_d - \dot{\mathbf{X}} = \mathbf{A}(\mathbf{X}_d - \mathbf{X}) + \mathbf{B}(\mathbf{U}^* - \mathbf{U}), \\ \mathbf{Y}_d - \mathbf{Y} = \mathbf{C}(\mathbf{X}_d - \mathbf{X}). \end{cases} \quad (9)$$

A state vector feedback control law of (11), designed to place stable poles in (10), stabilises (8), the delay-free system and consequently stabilises (1).

$$\mathbf{U} = \mathbf{K}(\mathbf{X}_d - \mathbf{X}) + \mathbf{U}^*, \quad (10)$$

$\mathbf{U}$  in (11) pushes the error,  $\mathbf{Y}_d - \mathbf{Y}$ , towards zero. (11) is an equivalent of (7).

**Enhancement 2 MIMO:**  $y_{di}(k + \max r_{ij} | j = 1 : n_u)$  is used to derive  $x_{di}$  and  $u_j$  for control law of (7).

As explained in the shortcoming (ii) of conventional MIMO SPCSs, including (5) and (6), a future output,  $y_{pi}$ , is predicted by Smith predictor. However, it is still compared with the present time reference,  $y_{ij}(k)$ , to produce the control error. As a result of this asynchrony, the predictive control system cannot track the reference appropriately. Enhancement 2 addresses this issue with use of future values of reference upfront to produce the control command.

The aforementioned tracking issue of conventional MIMO SPCSs is particularly apparent at the occasions of reference change. In order to eradicate the aforesaid tracking issue, the reference at time  $(\max r_{ij}) \times t_s$  ahead of current time is recommended to be used for control purposes.

### III. CASE STUDY, A MIMO CSTR

The case study is a complex exothermic continuous stirred tank reactor (CSTR) with two control outputs, three control inputs and three various delays. This model, (12), includes molar equilibrium and energy equilibrium equations, respectively. In fact, (12) is the combination of a single-delay two-output model presented in [18] and a double-input model of [19]; it also includes all possible delays, neglected in previous research. Two outputs of CSTR model of (12) are effluent concentration  $C_A(t)$  [mol/L] and temperature  $T(t)$  [K], where the control inputs are feed flow rate  $q_f(t)$  [L/min], water flow rate  $q_w(t)$  [L/min] and auxiliary temperature  $T_x(t)$  [K] (also known as coolant temperature, if it only cools down the outgoing fluid).  $\theta_1$ ,  $\theta_2$  and  $\theta_3$  [min] present time-delays.

$$\begin{cases} V \frac{dC_A(t)}{dt} = q_f(t - \theta_1)(C_f - C_A(t)) \\ + q_w(t - \theta_1)(0 - C_A(t)) - V\kappa \exp\left(\frac{-E}{RT(t)}\right) C_A(t), \\ V\rho c \frac{dT(t)}{dt} = (q_f + q_w)(t - \theta_2)\rho c(T_f - T(t)) \\ - VH\kappa \exp\left(\frac{-E}{RT(t)}\right) C_A(t) - U(T(t) - T_x(t - \theta_3)). \end{cases} \quad (11)$$

$V$ ,  $C_f$ ,  $\kappa$ ,  $E$ ,  $R$ ,  $c$ ,  $U$ ,  $H$  and  $\rho$  are reactor volume [L], feed concentration [mol/L], reaction velocity constant [ $\text{min}^{-1}$ ], Arrhenius activation energy [J/mol], gas constant [J/(mol.K)], specific heat capacitance [g/(L.K)], overall heat transfer coefficient [J/(min.K)], heat of reaction [J/min] and density [g/L], respectively. Water and feed temperature are assumed equal,  $T_f$ . All listed parameters are considered to be time-invariant.

#### A. Reduction of an Input and Linearization of the Model

This subsection explains how (12), with three inputs, is converted into dual models, each with two inputs: (i) a combination of (19) and (20) when concentration is increasing, and (ii) a combination of (19) and (21), when concentration is decreasing. These dual models were approximated by linear state equations of (22) and (23) for control system design purposes.

Feed and water valves, which are in charge of increase and decrease of concentration, should not work simultaneously. As a result, the effect of water flow can be replaced with its equivalent feed flow, as the sole control input, to reduce intricacy of the molar equilibrium equation and the whole model. In the first equation of (12), if the concentration is on rise  $q_w=0$  and can be eliminated; else, water flow rate of  $q_w$  is replaced by an equivalent negative flow of feed,  $q_f$ . The rate of mole number added to the tank with a flow of feed and water are  $q_f(C_f - C_A)$  and  $q_w(0 - C_A)$ , respectively. Thus, the right and the left side of (13) have the same effect on the fluid in the tank and are equivalent:

$$q_w = q_f \frac{C_A - C_f}{C_A}. \quad (12)$$

In other words, when the concentration is on decline, in the molar equilibrium equation,  $q_w$  can be replaced by its equivalent in (13); thus, only  $q_f$  remains in the first equation of (12), and one input,  $q_w$ , is eliminated from the model. The control system is then designed based on the resultant two-

input model. However, when a negative control command is generated for feed flow rate,  $q_f$ , water valve is actuated with the flow rate presented in (13), in practice.

If maximum flow rate of water valve is  $q_{w\max}$ , from (13), minimum algebraic  $q_f$  is calculated from (14):

$$q_{f\min} = -q_{w\max} \frac{C_A}{C_f - C_A}, \quad (13)$$

$$\text{or the range of } q_f = \left[ -\frac{C_A}{C_f - C_A} q_{w\max} \quad q_{f\max} \right]. \quad (14)$$

In the second equation of (12), energy equilibrium equation, at the time of concentration increase,  $q_w=0$ ; thus, it can be written as (16):

$$\begin{cases} V\rho c \frac{dT(t)}{dt} = q_f(t - \theta_2)\rho c(T_f - T(t)) \\ - VH\kappa \exp\left(\frac{-E}{RT(t)}\right) C_A(t) - U(T(t) - T_x(t - \theta_3)). \end{cases} \quad (15)$$

However, during concentration drop, feed flow is cut, and energy equilibrium equation can be written as (17):

$$\begin{cases} V\rho c \frac{dT(t)}{dt} = q_w(t - \theta_2)\rho c(T_f - T(t)) \\ - VH\kappa \exp\left(\frac{-E}{RT(t)}\right) C_A(t) - U(T(t) - T_x(t - \theta_3)). \end{cases} \quad (16)$$

With use of (13),  $q_w$  can be eliminated from (17):

$$\begin{cases} V\rho c \frac{dT(t)}{dt} = \frac{C_A - C_{Af}}{C_A} q_f(t - \theta_2)\rho c(T_f - T(t)) \\ - VH\kappa \exp\left(\frac{-E}{RT(t)}\right) C_A(t) - U(T(t) - T_x(t - \theta_3)). \end{cases} \quad (17)$$

Considering  $C_A$ ,  $T$ ,  $q_f$  and  $T_x$  as  $y_1$ ,  $y_2$ ,  $u_1$  and  $u_2$ , respectively. With use of nominal values of parameters listed in [18] and reasonable delays, molar equilibrium equation, the first equation of (12) is

$$\begin{cases} \dot{y}_1(t) = u_1(t - 0.05)(0.01 - 0.01y_1(t)) \\ - 7.2 \times 10^{10} y_1(t) \exp\left(\frac{-8750}{y_2(t)}\right), \end{cases} \quad (18)$$

Similarly, based on (16-18), if concentration increases/decreases, energy equilibrium equation, the second equation of (12), can be written as (20)/(21):

$$\begin{cases} \dot{y}_2(t) = u_1(t - 0.1)(3.5 - 0.01y_2(t)) + 1.506 \times \\ 10^{12} y_1(t) \exp\left(\frac{-8750}{y_2(t)}\right) - 2.092(y_2(t) - u_2(t - 0.2)). \end{cases} \quad (19)$$

$$\begin{cases} \dot{y}_2(t) = -\frac{y_1(t) - 1}{y_1(t)} u_1(t - 0.1)(3.5 - 0.01y_2(t)) + 1.506 \times \\ 10^{12} y_1(t) \exp\left(\frac{-8750}{y_2(t)}\right) - 2.092(y_2(t) - u_2(t - 0.2)). \end{cases} \quad (20)$$

For control system design purposes, combination of (19) and (20) was linearized around the operating point of  $\bar{y}_1 = 0.2$

mol/L and  $\bar{y}_2 = 310\text{ K}$ , to produce an approximate linear continuous model when the concentration is on rise:

$$\begin{bmatrix} \dot{y}_1(t) \\ \dot{y}_2(t) \end{bmatrix} = \begin{bmatrix} -0.0397 & 0 \\ 0.8310 & -2.0921 \end{bmatrix} \begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix} + \begin{bmatrix} 0.0080e^{-0.05s} & 0 \\ 0.4000e^{-0.1s} & 2.0921e^{-0.2s} \end{bmatrix} \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix}. \quad (21)$$

Similarly, if concentration is on decline, the approximate linear continuous model, out of (19 and 21), is

$$\begin{bmatrix} \dot{y}_1(t) \\ \dot{y}_2(t) \end{bmatrix} = \begin{bmatrix} -0.0397 & 0 \\ 0.8310 & -2.0921 \end{bmatrix} \begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix} + \begin{bmatrix} 0.0080e^{-0.05s} & 0 \\ -1.6000e^{-0.1s} & 2.0921e^{-0.2s} \end{bmatrix} \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix}. \quad (22)$$

### B. Control

In this research, the proposed enhanced MIMO SPCS was employed to simultaneously control concentration and temperature. Sole control of temperature [20] or concentration [18] have been reported for time-delay CSTR models. However, none of several double-input double-output control solutions for CSTRs (for example [21]) address models with time-delay(s). In order to develop the proposed enhanced SPCS, the following should be defined: (i) feedback control gains,  $\mathbf{K}$  in (11), (ii) a feedforward control law,  $\mathbf{U}^*$  in (11), (iii) maximum  $r_{ij}$ .

#### i) Feedback Control Gains

In this case study, the states of the systems are same as the outputs,  $y_1$  and  $y_2$ . Closed loop poles of -7 and -2 were opted for concentration and temperature,  $y_1$  and  $y_2$ , respectively. The pole related to concentration was chosen further from zero due to the higher importance of this output. With delay-free form of state space models, (22) and (23), arrays of feedback control gains,  $\mathbf{K}$  in (11), were found as (24) and (25) using pole placement technique detailed in subsection 10-2 of [22]. These realise aforesaid closed loop poles when concentration is on rise and on decline, respectively:

$$\mathbf{K} = \begin{bmatrix} 870.038 & 0 \\ -165.950 & -0.044 \end{bmatrix}. \quad (23)$$

$$\mathbf{K} = \begin{bmatrix} 870.038 & 0 \\ 665.786 & -0.044 \end{bmatrix}. \quad (24)$$

#### ii) Feedforward Control Law

From delay-free form of (22 and 23), for regulation, at the desired status of  $\dot{y}_1(t) = \dot{y}_2(t) = 0$ ,  $y_1 = y_{d1}$  and  $y_2 = y_{d2}$ , the following feedforward control law is derived:

$$\begin{cases} u_1^* = 4.9625y_{d1} \\ u_2^* = -1.3460y_{d1} + y_{d2}, & \text{for increasing concentration} \\ u_2^* = 3.3980y_{d1} + y_{d2}. & \text{for decreasing concentration} \end{cases} \quad (25)$$

#### iii) Maximum $r_{ij}$

Equations (22 and 23) can be converted to (27 and 28). (27)/(28) presents the system with increasing/decreasing concentration.

Knowing that a  $z^{-r}$  in a discrete model equals  $e^{-rt_s}$  in a continuous model in Laplace domain, max  $r_{1j}$  is  $0.05/t_s$  and max  $r_{2j}$  is  $0.2/t_s$ . For instance, with the sample time of  $t_s=1\text{ s}$  (sampling frequency of 1 Hz),  $y_{d1}$  and  $y_{d2}$  values at 0.05 (max  $r_{1j} \times t_s$ ) seconds and 0.2 (max  $r_{2j} \times t_s$ ) seconds ahead of simulation time were employed in (26) and for calculation of the states to be used in (7).

$$\begin{bmatrix} y_1(s) \\ y_2(s) \end{bmatrix} = \begin{bmatrix} \frac{0.008}{s+0.0397}e^{-0.05s} & 0 \\ \frac{0.4s+0.023}{s^2+2.132s+0.083}e^{-0.1s} & \frac{2.092}{s+2.092}e^{-0.2s} \end{bmatrix} \times \begin{bmatrix} u_1(s) \\ u_2(s) \end{bmatrix}. \quad (26)$$

$$\begin{bmatrix} y_1(s) \\ y_2(s) \end{bmatrix} = \begin{bmatrix} \frac{0.008}{s+0.0397}e^{-0.05s} & 0 \\ \frac{-1.6s-0.057}{s^2+2.132s+0.083}e^{-0.1s} & \frac{2.092}{s+2.092}e^{-0.2s} \end{bmatrix} \times \begin{bmatrix} u_1(s) \\ u_2(s) \end{bmatrix}. \quad (27)$$

## IV. RESULTS AND DISCUSSION

Figs. 1 and 2 show the simulation results for the proposed enhanced SPCS and a conventional SPCS with IMC feedback controllers, respectively. The original nonlinear system of (12) has been used in simulations with a sampling frequency of 1Hz, which is easily achievable in practice. The range of auxiliary temperature is [280 350] K. Random measurement noises of  $\pm 1^\circ\text{C}$  and  $\pm 0.002\text{ L/min}$  have been considered for temperature and concentration, respectively. Maximum flow rate of valves is 60 L/min. In order to be compared with the proposed enhanced SPCS, a conventional SPCS including IMC PI/PID feedback controllers were designed, partly similar to [23].

The proposed control system of (7)/(11), using parameters detailed in (24-26), as shown in Fig.1, resulted in both higher performance and less frequent rapid changes in actuation compared to IMC PI/PID feedback controllers, with results depicted in Fig.2. For the proposed control system, rapid changes of control inputs only happened at the beginning of operation or in the case of a change in the reference. Such a reasonably smooth change of control inputs is likely to be the influence of the feedforward component of the enhanced SPCS and agrees with the results and discussions of [24]. Such a feedforward component ideally hinders repeating deviations from the control equilibrium point as discussed in [24-26], which may happen with pure feedback control systems.

In Figs. 1 and 2, equivalent feed flow rate, as detailed in subsection III.A with the range of (15), has been presented. However, in practice, two separate valves spray water and high concentration feed into the reactor. Fig.3 demonstrates water and feed flows separately for the simulation presented in Fig.2.

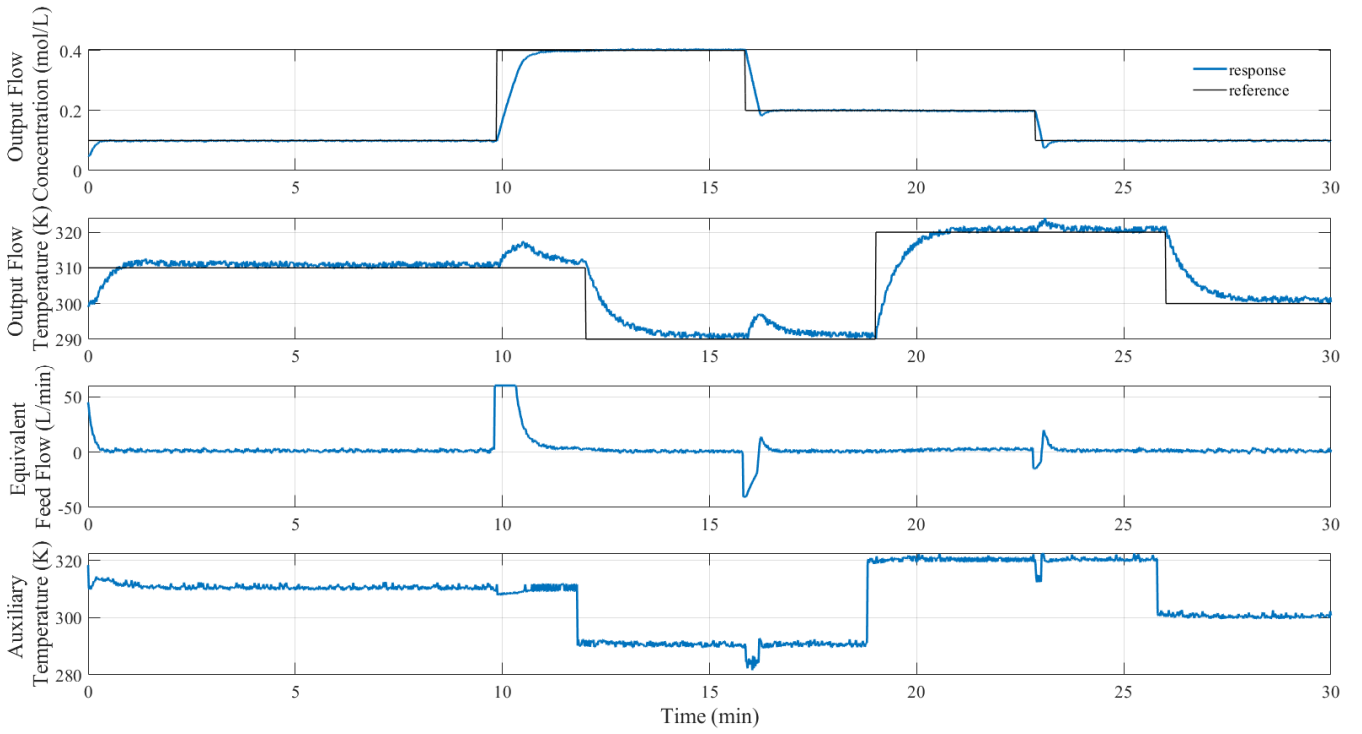


Fig. 1. Simulations results with the proposed enhanced Smith-predictor-based control approach with noises

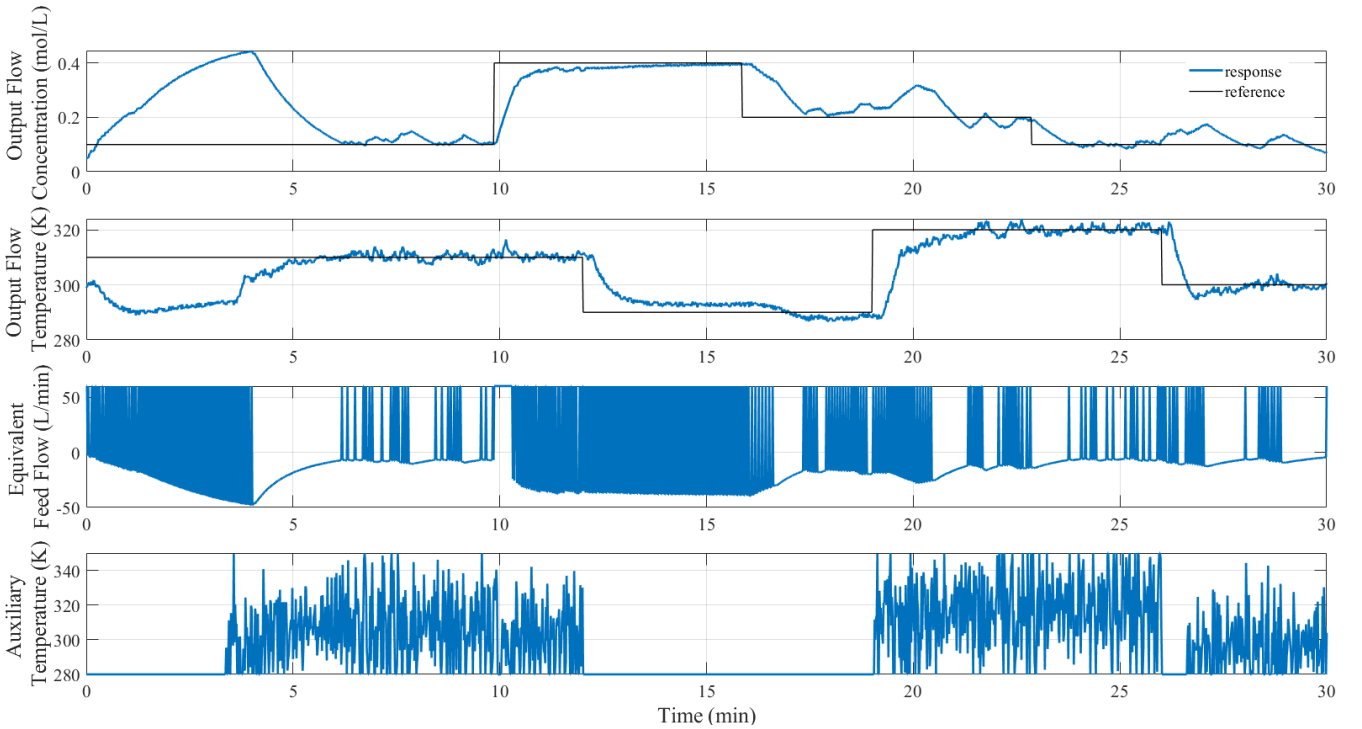


Fig. 2. Simulations results with conventional Smith-predictor-based control approach with noises

## V. CONCLUSION

In this paper, first, two shortcomings of the structure of conventional Smith-Predictor-based control systems were identified: (i) use of classical feedback controllers with their inherent constraints and (ii) asynchrony between the reference and the predicted output. The latter particularly causes the response to have a lag in tracking the reference. On this basis, two enhancements were proposed for SPCSs designed for MIMO time-delay systems: (i) replacing the array of classical feedback controllers by a feedback-

feedforward arrangement, and (ii) supplying some future reference value(s) upfront to the control system. Section II demonstrates the proposed control system in detail.

The proposed enhanced SPCS was tested to control a nonlinear exothermic MIMO CSTR and outperformed a conventional SPCS with IMC-based feedback controllers manifestly. The employed CSTR model includes unprecedented level of details and comprehensiveness in terms of inputs, outputs, time-delays, actuators' limits and sensor noises to assure that the proposed results are realistic, and the proposed method is implementable. The proposed

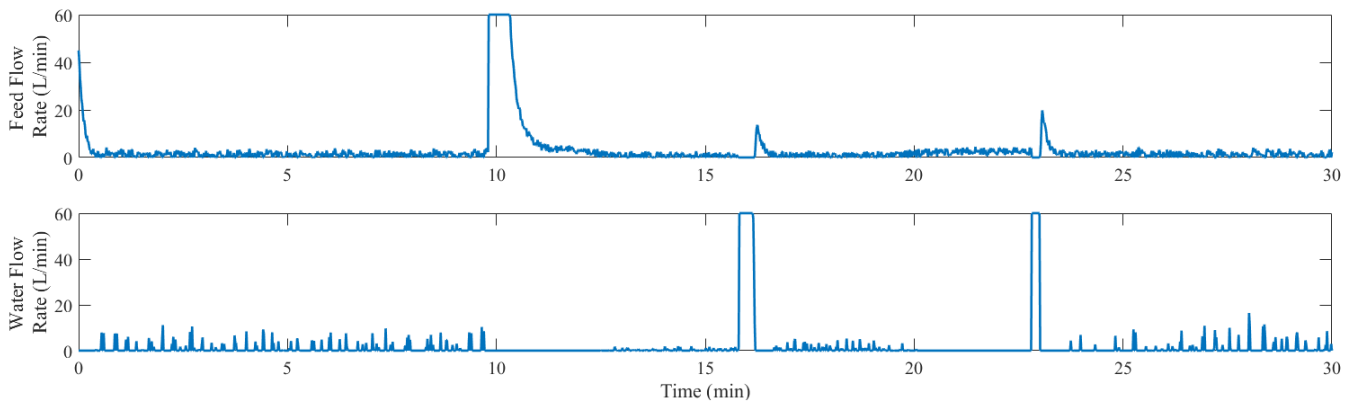


Fig. 3. Separate input feed and water flowrates for simulation presented in Fig.2

method not only presented an excellent tracking performance; but also, it witnessed much fewer rapid changes of control input. In fact, with use of the enhanced

SPCS, rapid changes of control input happened only at the beginning of operation or during reference change.

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