Smart Geotechnics for Smart Societies – Zhussupbekov, Sarsembayeva & Kaliakin (Eds)

The use of grading entropy in assessing granular soil hydraulic conductivity

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ABSTRACT: Hydraulic conductivity is an important geotechnical engineering property as it is linked to the performance of many geo-structures. This study used a recently compiled granular soil database (CG/KSAT/7/1278) to evaluate the best fit probability density functions for important parameters in the database. The Loglogistic function was found to be the best-fit probability density function for the parameters from CG/KSAT/7/1278 investigated in this paper. The grading entropy parameters (un-normalised) were used to develop a chart that highlights trends from the effect of changes of the gradation parameters on the estimated soil permeability.

1 INTRODUCTION

Hydraulic conductivity (k) is a fundamental soil mechanics parameter that is closely linked to many geotechnical problems, such as seepage, settlement, and slope stability (e.g. Lambe & Whitman 1969, Taylor 1948). Simple predictive models for k calibrated using laboratory data are often used (e.g. Chapuis 2012).

Feng (2022) assembled a coarse-grained soil database CG/KSAT/7/1278, which consists over 1200 measurements of k (see Feng 2022 and Feng et al. 2023 for full details of the assembled database including the original data-sources). Feng (2022) examined various empirical and semi-empirical transformation models for k of granular soils.

The aims of this study are: (a) Identify the best-fit PDFs for important parameters using CG/KSAT/7/1278; (b) Examine the influence of statistical outliers on the choice of best-fit PDFs from (a) and (c) Investigate if the grading entropy theory can offer insights into the variation of the soil permeability data from CG/KSAT/7/1278. For further details on the establishment and statistical analysis of the database see the thesis of Feng (2022) and Feng et al. (2023).

2 DATABASE STUDY

2.1 *Identifying potential outliers*

In this work the following procedure for identification of potential outliers was adopted: (i) datapoints with a computed standardized residual not within the range of -2 to 2 (cf. Montgomery et al. 2007), or those with a computed leverage value greater than three times the computed mean

DOI: 10.1201/9781003299127-28

leverage (cf. Velleman & Welsch 1981) were identified for all calibrated regression models (see Feng 2022 for a full list of the calibrated models); and (ii) if a datapoint from (i) was identified as an outlier for greater than 75% of the calibrated models it was deemed to be a statistical outlier. This analysis results in 22 datapoints out of 1278 in the CG/KSAT/7/1278 database (approximately 2%) being classified as statistical outliers (a similar procedure for identification of statistical outliers was adopted in Feng et al. 2021, Feng et al. 2022, and the procedure outlined in this paper is also used in Feng et al. 2023 for analysis of CG/KSAT/7/1278).

2.2 *Probability Density Functions (PDFs)*

The best fitting PDFs for the key parameters from the database CG/KSAT/7/1278 were evaluated using the Akaike Information Criterion (AIC) (Akaike 1974) before and after the removal of the statistical outliers as identified in Section 2.1. The AIC is computed as:

$$AIC = -2\log\left(L(\hat{\theta})\right) + 2K_i \tag{1}$$

where $L(\hat{\theta})$ is the 'likelihood function', and K_i is the number of parameters in the PDF fitted to the data. The trialled PDFs were: 'Lognormal', 'Exponential', 'Weibull', 'Loglogistic' and 'Gamma' (Feng and Vardanega 2019 carried out a similar study for a fine-grained soil k database: FG/KSAT-1358), were fitted to the void ratio (e), aperture diameter through which 50% of the material would pass (D_{50}) and k data from CG/KSAT/7/1278. In the following analysis the k data was analysed in the form of the intrinsic permeability K (length^2).

Figures 1–3 show probability plots where the aforementioned PDFs are fitted to the e, D_{50} and K data from CG/KSAT/7/1278 with and without the identified statistical outliers included. Tables 1 and 2 record the computed AIC for the PDFs trialled. The results show that the 'Loglogistic' function is the best-fit PDF for all parameter examined in database either before or after the removal of the identified statistical outliers. The 'Loglogistic' function can be expressed as (e.g. Johnson et al. 1994):

$$f(x|\mu,\sigma) = \frac{e^z}{\sigma x(1+e^z)^2}\ \text{, where } z = \frac{\log(x)-\mu}{\sigma} \tag{2}$$

where, μ = mean of the logarithmic values, and σ = scale parameter of the logarithmic values. Table 3 compares the fitted parameters of the best-fit PDFs ('Loglogistic') for the key parameters of the studied database with and without the identified statistical outliers. The effect of outliers on the fitted PDFs was deemed to be negligible.

Table 1. Computed AIC for the fitted PDFs for CG/KSAT/7/1278 (best fits shown in bold).

n =1278	Exponential	Lognormal	Weibull	Loglogistic	Gamma
e	1140	-628	-599	-801	-716
D_{50} (mm)	3810	2800	3313	2762	3524
K (mm^2)	-16736	-25291	-25108	-25388	-24269

Table 2. Computed AIC for the fitted PDFs for CG/KSAT/7/1278 with identified outliers removed (best fits shown in bold).

n =1256	Exponential	Lognormal	Weibull	Loglogistic	Gamma
e	1131	-716	-656	-880	-791
D_{50} (mm)	3582	2610	3127	2565	3331
K (mm^2)	-17532	-24883	-24701	-24972	-23946

Table 3. Best fit PDF (Equation 2) for key parameters of CG/KSAT/7/1278.

Fitted parameters	with the outliers included (n = 1278)			with outliers removed (n = 1256)		
	e	D_{50} (mm)	K (mm^2)	e	D_{50} (mm)	K (mm^2)
μ	-0.59	-0.71	-12.49	-0.59	-0.74	-12.48
σ	0.18	0.69	1.74	0.17	0.67	1.66

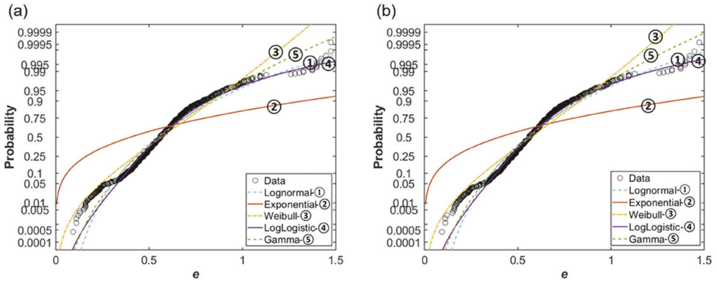

Figure 1. (a) Probability plot with PDFs fitted to data of e from CG/KSAT/7/1278 (n =1278), (b) Probability plot with PDFs fitted to data of e from CG/KSAT/7/1278 with identified statistical outliers removed (n =1256).

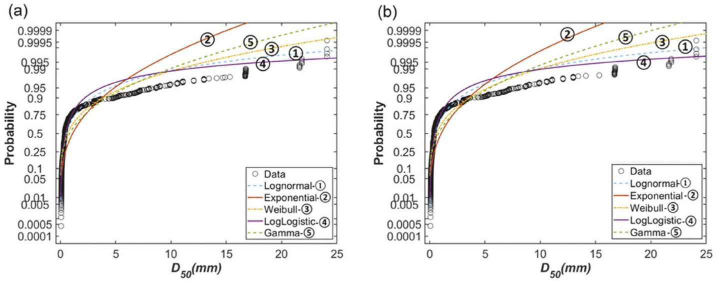

Figure 2. (a) Probability plot with PDFs fitted to data of D_{50} (mm) from CG/KSAT/7/1278 (n =1278), (b) Probability plot with PDFs fitted to data of D_{50} (mm) from CG/KSAT/7/1278 with identified statistical outliers removed (n =1256).

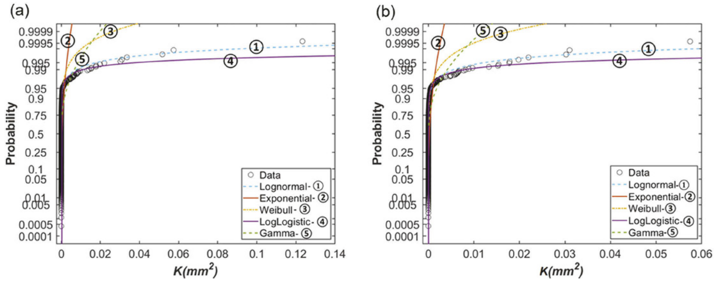

Figure 3. (a) Probability plot with PDFs fitted to data of K (mm^2) from CG/KSAT/7/1278 (n =1278), (b) Probability plot with PDFs fitted to data of K (mm^2) from CG/KSAT/7/1278 with identified statistical outliers removed (n =1256).

3 GRADING ENTROPY

The grading entropy approach was presented to investigate granular soil dry bulk density (e.g. Lőrincz 1990) and has subsequently been used to study a range of geotechnical problems (e.g. Lőrincz et al. 2005, Imre et al. 2009, Imre et al. 2012 and McDougall et al. 2013). Lőrincz et al. (2005) and Imre et al. (2009) give a detailed introduction of the framework (which characterizes disorder in soil particle size distributions) and full derivation of grading entropy parameters. The grading entropy S of a soil mixture is given by (form of the equations are shown as in Singh 2014, pp. 324-328):

$$S = S_0 + \Delta S \tag{3}$$

where S_0 = base entropy:

$$S_0 = \sum_{i=1}^{N} x_i \log_2 C_i \tag{4}$$

where N = number of fractions, x_i = relative frequency of fraction i, C_i = number of elementary statistical cells contained in fraction i. The entropy increment ΔS is:

$$\Delta S = -\sum_{i=1}^{N} x_i \log_2 x_i \tag{5}$$

the base entropy S_0 and the entropy increment ΔS helps explain the relative spread of the grain sizes and explains the relative distribution of the particles. The normalised grading entropy coordinates A and B are:

$$A = \frac{\sum_{i=1}^{N} x_i(i-1)}{N-1} \tag{6}$$

$$B = -\frac{\sum_{i=1}^{N} x_i \log_2 x_i}{\log N} \tag{7}$$

Feng et al. (2019) used the normalised grading entropy parameters (A, B) to assess the hydraulic conductivity for a granular soil mixture. O'Kelly & Nogal in their discussion of Feng et al. (2019) proposed the addition of the e alongside A and B for hydraulic conductivity prediction (Feng et al. 2020). O'Kelly & Nogal (2020) applied their three-parameter approach to test data on a wider range of granular materials than was presented in Feng et al. (2019, 2020). Feng et al. (2021) then investigated the influence of using the grading entropy parameter (S) along with the percentage air voids to assess asphalt concrete hydraulic conductivity using a large database. Imre et al. (2021) used the un-normalised grading entropy coordinates and determined iso-lines for the saturated k.

The grading entropy parameters for data from CG/KSAT/7/1278 were calculated with the width of the elementary cell (d_0) assumed to be 2^{-22}mm following Imre et al. (2009). Figures 4 and 5 present the normalised and non-normalized grading entropy diagram for the database CG/KSAT/7/1278. Figure 4 shows that the permeability levels exhibit an increasing trend with higher S_0 value and lower ΔS value. Such variation of permeability level is less distinct on the normalised grading entropy diagram (Figure 5). It was observed that datapoints with lower permeability level (i.e. categories 'very low', 'practically impermeable') mostly are aggregated around the A = 2/3 on the normalised grading entropy diagram, which according to Lőrincz (1990) represent mixtures with the maximum density for a given fraction number N, concerning a fixed distribution type (e.g., fractal distribution, non-fractal distribution).

4 CONCLUSIONS

Some statistical analysis of the coarse-grained soil database CG/KSAT/7/1278 has been presented in this paper. The main findings from this study are:

(a) the 'Loglogistic' function is the best fit PDF (of those trialled) for the e, D_{50} and K data from CG/KSAT/7/1278.

(b) The influence of the identified statistical outliers was found to be minimal on the best fit PDFs.

(c) On the normalised grading entropy diagram (A versus B), the K categories are less distinguishable than on the un-normalised grading entropy diagram (ΔS versus S_0) for the studied data.

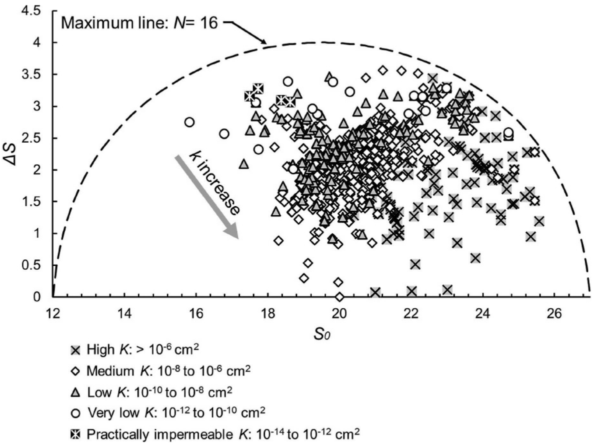

Figure 4. Grading entropy diagram; K categorization based on the categorization system given in Lambe & Whitman (1969, p. 288).

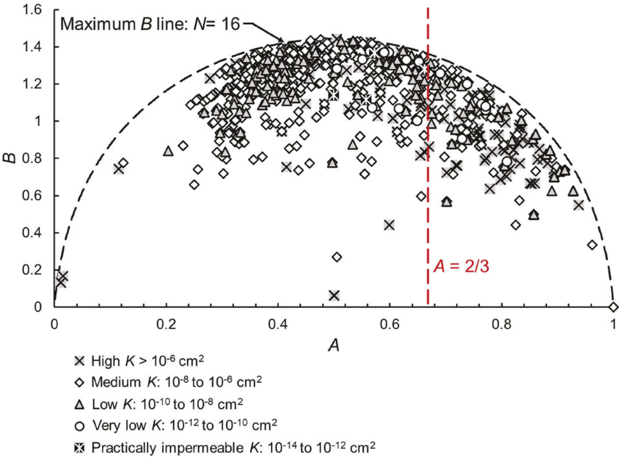

Figure 5. Normalised grading entropy diagram; K categorization based on the categorization system given in Lambe & Whitman (1969, p. 288).

ACKNOWLEDGEMENTS

SF acknowledges support from the China Scholarship Council (Grant Number 201708060067) for funding her doctoral studies at the University of Bristol. Data availability: This study has not generated new experimental data.

REFERENCES

Akaike, H. 1974. A new look at the statistical model identification. *IEEE Transactions on Automatic Control* 19(6): 716–723.

Chapuis, R.P. 2012. Predicting the saturated hydraulic conductivity of soils: A review. *Bulletin of Engineering Geology and the Environment* 71(3): 401–434.

Feng, S. & Vardanega, P.J. 2019. A database of saturated hydraulic conductivity of fine-grained soils: probability density functions. *Georisk: Assessment and Management of Risk for Engineered Systems and Geohazards* 13(4): 255–261.

Feng, S., Vardanega, P.J., Ibraim, E., Widyatmoko, I. & Ojum, C. 2019. Permeability assessment of some granular mixtures. *Géotechnique* 69(7): 646–654.

Feng, S., Vardanega, P.J., Ibraim, E., Widyatmoko I., Ojum, C., O'Kelly, B.C. & Nogal, M. 2020. Discussion: permeability assessment of some granular mixtures, *Géotechnique* 70(9): 845–847.

Feng, S., Vardanega, P.J., James, M. & Ibraim, E. 2021. Studying hydraulic conductivity of asphalt concrete using a database. *Transportation Engineering* 3: [100040].

Feng, S. 2022. *Hydraulic Conductivity of Road Construction Materials: with a focus on freeze-thaw effects.* PhD thesis, University of Bristol, Bristol, UK.

Feng, S., Ibraim, E. & Vardanega, P.J. 2022. Databases to assess the hydraulic conductivity of road construction materials. In M.M. Rahman & M. Jaksa (eds), *Proc. of Twentieth International Conference on Soil Mechanics and Geotechnical Engineering (ICSMGE 2022)*, *Sydney*, *1-5 May 2022*. Australian Geomechanics Society, Sydney, Australia, pp. 4547–4552.

Feng, S., Barreto, D., Imre, E., Ibraim, E. & Vardanega, P.J. 2023. Use of hydraulic radius to estimate the permeability of coarse-grained materials using a new geodatabase. *Transportation Geotechnics*, https://doi.org/10.1016/j.trgeo.2023.101026

Imre, E., Bálint, Á., Nagy, L., Lőrincz, J., Illés, Z., Barreto, D., Csini, F., Guida, G. & Feng, S. 2021. Examination of saturated hydraulic conductivity using grading curve functions. In *Proc. of 6th International Conference on Geotechnical and Geophysical Site Characterisation, Budapest*, 26-29 September 2021. https://doi.org/10.53243/ISC2020-373

Imre, E., Lőrincz, J., Szendefy, J., Trang, P.Q., Nagy, L., Singh, V.P. & Fityus, S. 2012. Case studies and benchmark examples for the use of grading entropy in geotechnics. *Entropy* 14(6): 1079–1102.

Imre, E., Lőrincz, J., Trang, Q.P., Fityus, S., Pusztai, J., Telekes, G. & Schanz, T. 2009. A general dry density law for sands. *KSCE Journal of Civil Engineering* 13(4): 257–272.

Johnson, N.L., Kotz, S. & Balakrishnan, N. 1994. *Continuous univariate distributions*. New York: John Wiley & Sons.

Lambe, T.W. & Whitman, R.V. 1969. *Soil Mechanics*. New York: John Wiley & Sons.

Lőrincz, J. 1990. Relationship between grading entropy and dry bulk density of granular soils. *Periodica Polytechnica Civil Engineering* 34(3): 255–265.

Lőrincz, J., Imre, E., Gálos, M., Trang, Q.P., Rajkai, K., Fityus, S. & Telekes, G. 2005. Grading entropy variation due to Soil crushing. *International Journal of Geomechanics* 5(4): 311–319.

McDougall, J.R., Imre, E., Barreto, D. & Kelly, D. 2013. Volumetric consequences of particle loss by grading entropy. *Géotechnique* 63(3): 262–266.

Montgomery, D.C., Runger, G.C. & Hubele, N.F. 2007. *Engineering Statistics* (4th ed.). New York: John Wiley & Sons Inc.

O'Kelly, B.C. & Nogal, M. 2020. Determination of soil permeability coefficient following an updated grading entropy approach. *Geotechnical Research* 7(1): 58–70.

Singh, V.P. 2014. *Entropy Theory in Hydraulic Engineering*. Reston: American Society of Civil Engineers.

Taylor, D.W. 1948. *Fundamentals of Soil Mechanics*. New York: John Wiley & Sons.

Velleman, P.F. & Welsch, R.E. 1981. Efficient Computing of Regression Diagnostics. *The American Statistician* 35(4): 234–242.