

Demand Weighted Edge Betweenness Centrality for Identifying Critical Pipes in Water Distribution Networks

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Abstract: Water distribution networks (WDNs) support urban resilience by delivering potable water under highly variable demands and hydraulic conditions. Traditional graph-theoretic centrality measures identify critical pipes based solely on topology, overlooking the spatial heterogeneity of nodal demands that drives service loss priorities. This study introduces Demand Weighted Edge Betweenness Centrality (DWEBC), a novel metric that integrates pipe length, head loss, and target-node demand into edge-weight calculations, thereby aligning shortest-path computations with hydraulic relevance of the paths. Three weighting schemes; global constant, time-varying nodal, and time-varying nodal and pipe-specific scaling, are formulated and implemented within a Python/WNTR environment with NetworkX handling graphs. The algorithm is validated on a small laboratory-scale Kaina network and the medium-size Exnet benchmark and it demonstrates that DWEBC, in particular the time-varying nodal and pipe-specific variant, distinctly ranks pipes that serve major demand corridors. This offers rapid screening tool for maintenance prioritization and emergency planning. Computational performance outruns several ‘remove and simulate’ approaches by orders of magnitude, enabling routine re-analysis under shifting demand scenarios. By combining global connectivity insights with operational data, DWEBC provides network operators with transparent, actionable intelligence for capital-planning and resilience assessment, and can aid leak detection. In addition to being interpretable, it is easily scalable and can make use of real-time flow telemetry to enable dynamic and hybrid frameworks such as Digital Twin (DT) to enable real-time and realist network performance analysis and control.

Water distribution networks (WDNs) underpin urban resilience by sustaining potable water delivery under dynamically varying demands and hydraulic states. Conventional graph-theoretic centrality measures assess component importance purely from network topology, neglecting spatially heterogeneous demand patterns and hydraulic losses that dictate service degradation priorities. This study introduces *Demand Weighted Edge Betweenness Centrality* (DWEBC), a hydraulically informed centrality metric that incorporates pipe length, head loss, and nodal demand into edge-weighting schemes, thereby reorienting shortest-path computations toward hydraulically relevant flow corridors. Three formulations are proposed: (i) global constant weighting, (ii) time-varying nodal weighting, and (iii) time-varying nodal with pipe-specific scaling. The methodology is implemented within the Python/WNTR environment, with NetworkX providing graph analysis functionality. Application to the laboratory-scale Kaina network and the benchmark Exnet system demonstrates that DWEBC, particularly the time-varying nodal and pipe-specific variant, effectively differentiates pipes that serve dominant demand corridors, outperforming purely topological measures in reflecting hydraulic criticality. Relative to computationally expensive “remove-and-simulate” reliability analyses, DWEBC achieves orders-of-magnitude improvements in runtime, enabling routine reanalysis across shifting demand and operating scenarios. The metric thus provides a scalable

and interpretable screening tool for critical pipe identification, with direct implications for preventive maintenance prioritization, emergency response planning, and leak localization. Moreover, its compatibility with supervisory control and data acquisition (SCADA) and real-time telemetry facilitates integration within Digital Twin frameworks, enabling dynamic performance tracking, resilience assessment, and adaptive operational control under evolving urban water supply conditions.

Author Keywords: Demand Weighted Edge Betweenness Centrality (DWEBC); Hydraulic Criticality; Network Resilience; Graph-Theory.

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Introduction

Water distribution networks (WDNs) form the lifeline of urban infrastructure, delivering potable water to millions of households, businesses, and critical facilities. They are complex and dynamic infrastructures that play this vital role of providing clean and safe water to communities, institutions and industries, at the lowest cost possible (Acree et al., 2024; Fan et al., 2021; Kowalski et al., 2015; Lorenz et al., 2021). As such, the management of WDNs is crucial for delivering safe and reliable drinking water, yet aging pipes, shifting demand patterns, and occasional extreme events undermine system resilience, leading to costly bursts and service interruptions. A single pipe break, detected or undetected, can not only interrupt service for thousands of consumers but also induce inefficiencies that result in pressure instability, water wastage, increased energy consumption, and pose public-health risks through contaminant intrusion and property damage from flooding (Berardi et al., 2021; Huzsvár et al., 2019). This is compounded by the fact that water supply in urban areas necessitates additional pumping to satisfy pressure and demand requirements due to terrain, increased demand, and high-rise buildings, and that there is stricter regulations on water quality and water loss (Ulusoy et al., 2022). Additionally, traditional methods for monitoring, leak detection, and optimizing operations in such systems can be time-consuming, labour-intensive, and computationally expensive.

The importance of efficiency in WDNs has been a subject of research for decades, with the goal being the provision of reliable water supply, reducing water loss, and optimizing operation (Campbell et al., 2015; Khoa Bui et al., 2020). However, this has been a difficult undertaking due to the complexity of WDNs resulting from urbanisation, necessitating partitioning into manageable zones on which leakage, pressure management and maintenance can be effective (Campbell et al., 2015; Giustolisi and Ridolfi, 2014; Morrison et al., 2007; Savić and Ferrari, 2014). This is beneficial as the small zones known as District Metered Areas (DMAs) pose a reduced handling problem on which the parameters are monitored and controlled (Sharma et al., 2022). Partitioning a WDN into DMAs falls into two major phases, namely the clustering algorithms: identification of numbers of feasible DMAs by identifying communities on which clustering algorithms are applied to group network nodes in the WDN

based on similarity metrics that reflect relevant hydraulic or operational characteristics, and the sectorisation procedures: physically separating the network by installing valves and/or flow meters at strategic positions identified through graph network analysis such as graph theory(Creaco et al., 2023; Giudicianni et al., 2020; Sharma et al., 2022).

There have been many studies on WDN partitioning and control and critical pipes identification, from rigid DMAs with permanent boundaries to design for control attempts(Sharma et al., 2022; Ulusoy et al., 2022). As (Khoa Bui et al., 2020) notes, most of the existing clustering algorithms and critical pipes identification algorithms for WDNs relate to Graph Theory(GT). By converting the WDN into a graph, GT properties can guide partitioning and resilience analysis processes. GT is a mathematical analysis tool based on the idea that connections in a system can be modelled as a graph and therefore be used to analyse their relation to the functions of the system(Acree et al., 2024). A graph is a pair of sets $G = (V, E)$ where V represents the vertices (nodes or junctions) and E represents edges(links or pipes or pumps)(Schaeffer, 2007; Shekofteh et al., 2023). The use of GT for partitioning involves establishing communities within the network, in which vertices represent demand nodes, tanks, and reservoirs, while edges represent pipes, pumps, and valves. Several algorithms have been applied for partitioning and identifying critical corridors in a WDN using GT with the goal being connectivity within the network. The depth-first search (DFS) and breadth-first search (BFS) use an exploration approach that tracks nodes to establish connectivity between nodes that have been visited. DFS and BFS were used on a network in Mexico to identify nodes belonging to sectors and examine configurations of sectors and their size and node linkages(Tzatchkov et al., 2006). Hence stronger connections could be identified between nodes, as exemplified by the independent works of Ostfeld and Lifshitz and Ostfeld(Khoa Bui et al., 2020; Perelman and Ostfeld, 2011). A Depth-first search has also been used to partition a single source and multiple source partitioning of a WDN(Tzatchkov and Alcocer-Yamanaka, 2019). Others have used these algorithms combined with others in sector identification. The DFS, BFS and the algorithm for finding the shortest path(Dijkstra algorithm) were used to automatically create DMAs in a WDN(Alvisi and Franchini, 2014). There has also been work on using the Floyd-Warshall algorithm to partitioning a WDN into DMAs(Gomes et al., 2012).

As noticed, one crucial property is connectivity, which refers to the number of independent paths available between any two nodes in the network. In fact the parameters for assessing network connectivity can be used to assess WDN reliability(Pagano et al., 2019; Trifunović, 2012). High connectivity indicates a robust network that can withstand disruptions in specific locations without compromising water delivery to other areas(Ulusoy et al., 2022). Conversely, low connectivity suggests a more fragile network where a single pipe failure could isolate entire sections. That's the graph-theoretic centrality measures are used as rapid, surrogate indicators of critical elements in network analysis, which assigns values to nodes and edges that represent their importance in the network(Brandes, 2008; Ugurlu, 2022). There are four types of centrality measures that have been widely explored, namely Degree Centrality(DC) which identifies the most connected node as the most central node, Leverage Centrality(LC) which compares a node with its neighbours, closeness centrality (CC) which identifies a node as being central if it can reach its neighbours in fewer steps, and Betweenness Centrality (BC) which is

one of the most popular measure that is based on the shortest path principle (Brandes, 2008; Ugurlu, 2022). BC defines a node/edge as being central if it has many shortest paths between pairs of nodes passing through it (Seyed Ashkan Zarghami, 2018), and is the focus of this work.

While BC is widely used in identifying and assessing central nodes (junctions) and edges (pipes) (Bergamini and Meyerhenke, 2016; Brandes, 2008; Seyed Ashkan Zarghami, 2018; Ugurlu, 2022) and defining the partitions, it has several limitations when applied to WDNs owing its focus on topology and shortest path principle. By definition, it calculates the number of shortest paths passing through a node, however water flow paths, just like electric current and vehicles in traffic, are not equal in terms of service consequence (Newman, 2005; Ulusoy et al., 2018), for Hydraulic principles and parameters such as demand, pressure, and pipe diameters affect the flow paths and operation of the WDN. The traditional BC is applied to a static WDN while WDNs are dynamic in nature as flow changes based on demand, head loss, operational controls as well as pressure, making the purely topological metric BC less representative of the network. There has been works to incorporate such parameters. For example, Ulosoy et al (2018) employed BC with hydraulic information in measuring the criticality of pipes in a WDN. They applied their method on a case study to estimate the resilience of a network (Ulusoy et al., 2018). In their study, a measure was developed, termed Water Flow Edge Between Centrality (WFEBC), that integrated energy redundancy into the graph theoretic measures of betweenness centrality. However, the none-to-node paths were treated equally in their work. (Meijer et al., 2021) developed a methodology that used leak location to identify critical pipes in a WDN. Another study focused on the vulnerability of WDN using BC and information entropy (Seyed Ashkan Zarghami, 2018). While Simone (Antonietta Simone, 2018) considered edge betweenness in tailoring centrality measures to WDNs (2018). (Hajibabaei et al., 2024) applied edge betweenness centrality to the design of WDN by using a dynamic weighting function to the edges to determine the pipe diameters in a single source WDN. (Agathokleous et al., 2017) used betweenness centrality to demonstrate the effect of topology on WDN. Recently, BC was applied to a case study to evaluate the hydraulic behaviour of the network and identify main water supply paths in a WDN (Ciliberti et al., 2023), while (Satish et al., 2024) considered pipe failure combinations in a 'remove and simulate' approach.

However, a clear gap remains as the explored metrics, while some involved flow in the edge weights, they ignored spatial heterogeneity in demand for no existing formulation of edge betweenness simultaneously accounts for both origin and destination demands contribution on the path: a pipe that sits on many shortest paths between two low-demand junctions can score as highly as one that connects major supply and demand centres, even though the latter's failure would inflict far greater service loss. This work hence to ensure that the BC reflects the physical and operation conditions and is hydraulically relevant, explores extending the Brandes's algorithm by using the demand at the target nodes in the edge weights calculations for the edge betweenness centrality. The metric can highlight pipes whose failure would sever high-demand corridors, even if they lie on less-travelled topological backbones. The work calls this metric the Demand Weighted Edge Betweenness Centrality (DWEBC). The metric is formulated and implemented in Python using Water Network Tool for Resilience (WNTR). We validate

DWEBC on both laboratory network in Harbin, China, and Exnet benchmark network, comparing its performance and reliability against recent methods in literature for the same purpose. Finally, we discuss practical considerations for utility adoption, and integration into GIS/SCADA platforms, and recommended workflows, demonstrating how DWEBC can serve as a rapid first-pass tool for prioritizing pipe inspections, maintenance, and emergency response planning. The DWEBC approach is new in that its edge weights are calculated by considering demand, head loss and geodesic paths in determining the criticality of edges. The metric offers a computationally efficient, globally informed ranking that is responsive to both network structure and demand distribution, and it fills the identified gap and ensures more responsive resilience analytics.

Methodology

Graph Theory in WDN

The widely used BC ranks the importance of nodes based on their participation in the shortest paths of the network. We model the WDN as an undirected (since direction may change during operation conditions like failure conditions or valve operation) graph $G = (V, E)$ to represent a WDN with n nodes and m edges, modelled also to incorporate nodal demand and pipe capacity. In WDN, for $n \in V$ there exists at least one edge to another node. Naming σ_{st} the number of shortest paths from a node s to a node t and $\sigma_{st}(v)$, the number of shortest paths from s to t that go through edge v , the classical edge betweenness centrality of a given edge v is defined as:

$$C_B(v) = \sum_{s,t \in V, s \neq t \neq v} \frac{\sigma_{st}(v)}{\sigma_{st}} \quad (1)$$

Considering all the shortest paths in the WDN, the $C_B(v)$ represents the fraction of shortest paths that pass through that edge v . The understanding is that nodes with high betweenness can be important in partitioning a WDN because they identify nodes and pipes that influence WDN efficiency and resilience. The betweenness centrality is determined by two steps:

1. Computing the length and number of shortest paths between all pairs of nodes
2. Sum all pair-dependencies

Demand Weighted Edge Betweenness Centrality

Central to our approach is the adaptation of Brandes's algorithm for betweenness centrality to include demand weighting. Demand Weighted Edge Betweenness Centrality (DWEBC) extends beyond just quantifying (Ulusoy et al., 2018) the number of times an edge acts as a pipe along the shortest path between nodes in the network. Incorporating node-specific hydraulic parameters, specifically demand, in the weighting function ensures that pipes/edges to nodes with higher demands are prioritised, reflecting their relevance in the WDN. The conventional BC will not integrate the varying demand across nodes thus overlooking critical areas that may be critical for the WDN connectivity and overall performance. DWEBC enables more accurate identification of critical edges for such purposes as WDN partitioning and reliability assessments by addressing the limitations of traditional BC and other measures in incorporating demand data.

Since simulating the consequences all failure scenarios is prohibitive both computationally and combinatorially, a common approach is to isolate pipe failures. This procedure involves ranking the pipes in terms of their relevance in the WDN. While water flows to all available paths, the critical or relevant paths are those leading to high-demand nodes. The adjacency matrix is computed by coefficients obtained from the demand and head losses for the nodes and pipes as shown in Equation 2. We propose hence a function whose output depends on the relevance of the target node, in this case the magnitude of demand. The approach extends the procedure introduced by (Giustolisi et al., 2020) in embedding the intrinsic strength of the nodes.

We represent the network as a weighted graph, which contains the cost (weight) for the edge $s-t$. Suppose we have n nodes, we hence define the $n \times n$ matrix W such that:

$$W_{s,t} = \begin{cases} w_{(s,t)} & \text{if there is a directed edge from } s \text{ to } t, \\ \infty & \text{otherwise} \end{cases} \quad (2)$$

As such the diagonal matrix is constructed such that each element corresponds to the sum of weights ($w_{(s,t)}$) of incoming edges. In this we will have captured the attractive force of target nodes.

$$D_{tt} = \sum_s W_{st} \text{ that's for incoming weights to this node}$$

The combinatorial graph Laplacian will be defined as:

$$L = D - W$$

With its entries being

$$L_{st} = \begin{cases} D_{st} & \text{if } s = t \\ -W_{st} & \text{if } s \neq t \end{cases}$$

With this formulation, the weight function represents how less attractive the connection is, in which an edge from s to t with low l_{st} and a high $d(t)$ will be highly attractive. The diagonal entry $D_{tt} = \sum_s W_{st}$ represents the incoming attraction of the node. Hence the Graph Laplacian embodies the difference between the connection strength to other nodes and the attractiveness of the node.

The DWEBC is formulated by modifying the BC measure and presented as Equation (1). First, we obtain the edge weighting function $w_{(s,t)}$ that includes nodal demands and pipe lengths as follows for an edge(pipe) from node s to node t :

$$w_{(s,t)} = \alpha l_{(s,t)} \times \frac{1}{d(t) + \epsilon} \quad (3)$$

where $l_{(s,t)}$ is the pipe length between demand nodes s and t , $d(t)$ is the base demand at the target node t , ϵ is a small constant to avoid division by zero, and α is a global scaling factor calibrated using observed data: typical head loss across an edge, pipe length, and demand.

In the weighting function, the scaling factor α controls the relative importance of the demand compared to the pipe length. This has been modelled to be in the order of observed (computed head loss). The scaling factor calibrates the weighting function to a physically meaningful

range, as revealed through dimension analysis, one that is comparable to the head losses as it has the dimensions of volumetric flow rate.

By inversely weighting demand in the denominator, high-demand nodes incur a lower traversal cost; shortest-path algorithms will thus preferentially route through corridors serving large demands, aligning path enumeration with service-loss priorities. The betweenness centrality interprets weight as an edge cost; hence shorter pipe length will be preferred. In the same understanding, a path with a lower target node demand, with a lower cost, will be regarded as considered as a high costs path, which is against the goal of what we are trying to achieve, for in WDN higher demands should generally have higher centrality and are critical in the network. This way, low-demand nodes (with high inverse demand) will appear more central in the network, while nodes with high demand (low inverse demand) will appear less central. The work considers three cases that investigate the hydraulic sensitivity of weighting function in defining critical pipes.

Case 1: For the case of global constant α shown in **Equation 3**, given the average nodal demand d_{avg} average pipe length l_{avg} , and average head loss h_f between nodes, it is calculated as follows:

$$\alpha = h_f \times \frac{d_{avg}}{l_{avg}} \quad (4)$$

In this case, all the terms are averaged over the entire network and over all timesteps for extended period simulation (EPS).

Case 2: For the complicated case in which the α is not constant, i.e. the scaling factor is node-specific, the weighting factor is modelled as:

$$w_{(s,t)} = \alpha_{(s,t)} l_{avg} \times \frac{1}{d(t) + \epsilon} \quad (5)$$

This case would be valuable in instances in which the WDN is sensitive to nodal demand due to such factors as widely varying demand patterns and pipe materials, with uniform pipe lengths. In this scenario, the variable $(\alpha_{(s,t)})$ is defined as:

$$\alpha_{(s,t)} = h_{f(s,t)} \times \frac{d_{(s,t)}}{l_{avg}} \quad (6)$$

In **Equation 6** for the variable scaling factor, the terms $h_{f(s,t)}$ and $d_{(s,t)}$ are computed for the pair of nodes concerned and averaged over the timesteps for EPS.

Case 3: For this case α is also not constant, i.e. the scaling factor is both node and pipe-specific, the weighting factor is modelled as:

$$w_{(s,t)} = \alpha_{(s,t)} l_{(s,t)} \times \frac{1}{d(t) + \epsilon} \quad (7)$$

This case would be valuable in networks with heterogeneous materials or conditions, in which the WDN is overly sensitive to nodal demand and pipe material as when we have widely

varying demand patterns and profiles, and the network has wide varying pipe materials as opposite to lengths between nodes. In this scenario, the variable ($\alpha_{(s,t)}$) is defined as:

$$\alpha_{(s,t)} = h_{f(s,t)} \times \frac{d_{(s,t)}}{l_{(s,t)}} \quad (8)$$

In **Equation 8** for the variable scaling factor, the terms $h_{f(s,t)}$, $l_{(s,t)}$, and $d_{(s,t)}$ are computed for the pair of nodes concerned and $h_{f(s,t)}$ and $d_{(s,t)}$ not averaged over the timesteps for EPS.

The computation for the three cases follows the procedure stipulated in in **Algorithm 1**.

Brandes' algorithm, with Dijkstra's algorithm for weighted graphs, is used in computing the shortest paths between nodes. We then have, using the edge weights ($w_{(s,t)}$), the demand weighted edge betweenness centrality for edge v as:

$$DWBC_v = \sum_{\substack{s,v,t \in V \\ s \neq v \neq t}} \frac{\sigma_{st}(v)}{\sigma_{st}} \quad (9)$$

The overall flow in the WDN is influenced by the high-demand nodes, as prioritised by the weighting function, and nodes along the preferred paths, even if they are not high-demand nodes themselves, will acquire proportionally high betweenness centrality. Thus, with the DWBC, nodes that are key to the network's connectivity but also function as bridge to many or high-demand customers are identified. The DWBC will also reveal nodes to be prioritized during maintenance and resilience enhancements. As would be most obvious, pipes and pumps that connect high DWBC nodes will require larger capacities or higher performance to ensure peak demand is met, hence DWBC allows for targeted investment. Network expansions can also benefit from this metric as critical nodes will be identified, and pipeline routing optimised simultaneously. Water Loss Reduction efforts can benefit from this metric by prioritising the nodes with high DWBC for leak detection as it would cause significant water loss and disrupt major consumers.

Algorithm 1: Demand Weighted Edge Betweenness Centrality (DWBC)

Inputs: Epanet file

- res_link: list of reservoir-connected pipe names
- eq: choice of α/W calculation (1, 2, or 3)
- epsilon: small constant to avoid division by zero

Outputs: For each timestep $t=0 \dots 23$, betweenness centrality (with weights w_1, w_2, w_3)

1. Load & Preprocess

```

wn ← WaterNetworkModel(file)
For each pipe p in wn.pipe_name_list: p.length ← p.length / 1000
ps ← [p for p in wn.pipe_name_list if p ∉ res_link]
s_t ← [(p.start_node, p.end_node) for p in ps] # initial (s,t) pairs

```

2. Run Hydraulic Simulation

```

Set options' (demand_model, pressures, duration, timesteps)
results ← EpanetSimulator(wn).run_sim()
flow, headloss ← results.link[['flowrate','headloss']][:24]
demand ← results.node.demand[wn.junctions][:24]

```

3. Get True Flow Directions per Timestep

```

function get_true_st(flow, s_t):

```

```

drop reservoir columns from flow
For each timestep T:
    Identify pipes with flow < 0
    Flip (s,t)→(t,s) for those pipes
    Append adjusted s_t[t] to s_t_list
return s_t_list
s_t_list ← get_true_st(flow, s_t)

4. Build Time-Indexed Node Lists
function get_t(s_t_list):
    For each s_t in s_t_list:
        t_nodes[t] ← [end_node for (start_node,end_node) in s_t]
    return t_nodes

t_list ← get_t(s_t_list)

5. Compute  $\alpha$  (alpha)
function get_alpha(headloss, demand, s_t_list, eq):
    Filter out reservoir pipes from headloss
    switch eq:
        case 1:  $\alpha \leftarrow \text{mean}(\text{headloss}) * \text{mean}(\text{demand}) / \text{mean}(L)$ 
        case 2:  $\alpha[t] \leftarrow \text{mean}(\text{headloss}[t]) * \text{mean}(\text{demand}[t]) / \text{mean}(L)$ 
        case 3:  $\alpha[t,i] \leftarrow \text{headloss}[t,i] * D'[t,i] / L[i]$ 

6. Compute W (weights)
function get_W(headloss, demand, s_t_list, eq, epsilon):
    switch eq:
        case 1:  $W[t,i] \leftarrow \alpha * L[i] / (D'[t,i] + \epsilon)$ 
        case 2:  $W[t,i] \leftarrow \alpha[t] * L[i] / (D'[t,i] + \epsilon)$ 
        case 3:  $W[t,i] \leftarrow \alpha[t,i] * L[i] / (D'[t,i] + \epsilon)$ 
    return W

7. Build Graphs and compute DWEBC
pos ← {n: wn.get_node(n).coords}
for each t=0...23:
    G ← DiGraph()
    for i,(u,v) in enumerate(dirs[t]):
        G.add_edge(u,v, flow=flow[t,i], hl=headloss[t,i], length=L[i], w1=W1[t,i], w2=W2[t,i], w3=W3[t,i])
    for w in [w1, w2, w3]:
        Call Brandes' and Dijkstra's Algorithms
    dwebc ← betweenness centrality(G, weight=w) #normalised
Result: DWEBC
End.

```

Case Studies

The proposed edge weighting strategy was applied to two case studies: Kaina, a laboratory network in Harbin, China and Exnet Network, a benchmark network.

Kaina Case Study

Kaina Network, presented in **Fig. 1**, is an experimental lab-sized water distribution network. It consists of 79 junctions, 3 reservoirs, 123 pipes, and 2 pumps. The network is designed for the study of network partitioning and secondary water supply system (SWSS) control. Considering the goal of the study, all existing valves were removed and during start nodes and end nodes

calculations the pumps are not included. The network partitioning has three levels and for purpose of this study, the network is loaded only in level three with a base demand of 0.5LPS, and a demand pattern given in given in **Fig 2**. All nodes have an elevation of 0m, except for reservoir nodes R-1(2m), R-2(2m), and R-3(20m).

The WNTR toolset was used to import the network shown in **Fig. 3**, and using steps given in **Algorithm 1**: a list of pipes was obtained (excluding reservoir links and converting the respective lengths to meters as EPANET stores lengths in meters), and a set of list of start and end nodes of pipes was also obtained for list of pipes.

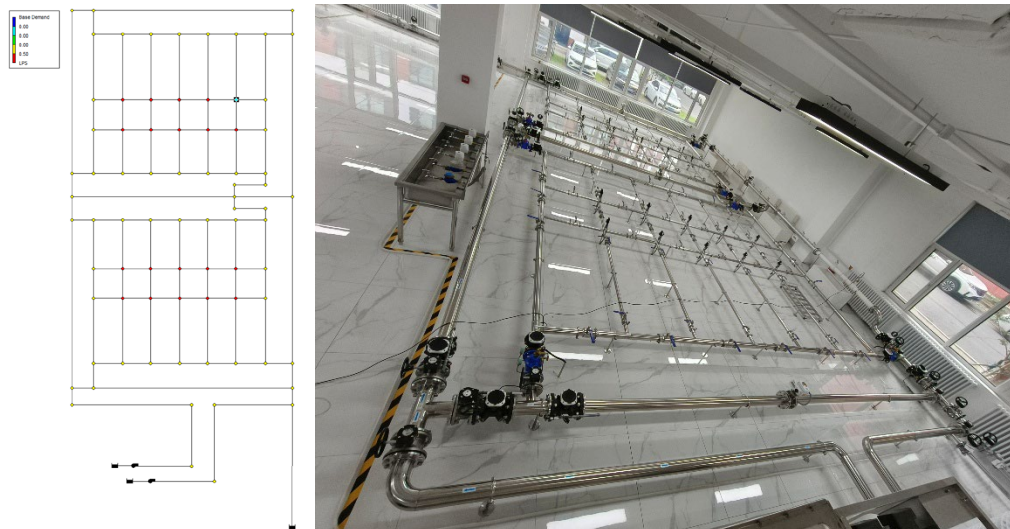


Fig. 1. Kaina Laboratory Experimental Network (Left: network; Right: Network photo).

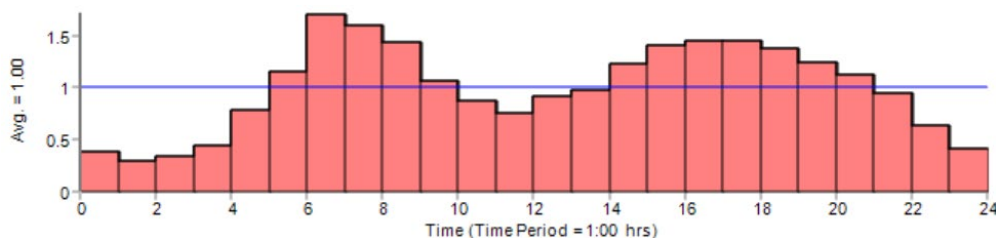


Fig. 2. Demand Pattern used.

The pipes directly connected to sources (reservoirs or tanks) are identified to exclude them from connectivity and weighting computations. The hydraulic options used the Pressure-Dependent Demand (PDD) model, setting the required and minimum pressures, and run time as shown in **Table 2** below.

Table 2. Hydraulic Simulation options

Parameter	Setting
Demand Model	PDD
Hydraulic Required Pressure	10
Hydraulic Minimum Pressure	3
Time Duration	24*3600
Hydraulic Timestep	3600
Report Timestep = 3600	

The EPANET simulator was run and for the first 24 hrs the link flows and head loss, and node demands were obtained, leaving out reservoir pipes whenever we need per-pipe computations. Since EPANET may report negative flows (indicating the “start” and “end” nodes are reversed), we correct each pipe’s direction for each timestep by obtaining pipes with flow less than zero and swapping the respective (start, end) for the pipe to return a corrected direction list for start and end nodes that define a pipe. This allows us to now have a correct downstream node for each pipe for which we can rightly extract the nodal demand at the pipe outlet for each timestep.

After getting the true flow directions and aligning the demands to pipes, the scaling factor is computed for *cases 1 to 3*, and the weight matrices W for each case is built as below:

$$W[t, i] = \begin{cases} w1 = \frac{\alpha L_i}{D'[t, i] + \varepsilon}, & \text{case 1} \\ w2 = \frac{\alpha[t] L_i}{D'[t, i] + \varepsilon}, & \text{case 2} \\ w3 = \frac{\alpha[t, i] L_i}{D'[t, i] + \varepsilon}, & \text{case 3} \end{cases} \quad (9)$$

Finally, we can construct the Graph using NetworkX, adding an edge for each pipe i with pipe name, flow, head loss, length, and weight attributes obtained. The DWEBC was computed for each weight variant through Brandes’ algorithm and using Dijkstra to handle weights.

The algorithm produced varying values for the DWEBC for the different timesteps for the different scaling factor and weight values. The results for timestep 2 for all three cases have been plotted in **Fig. 3** to demonstrate the different DWEBC values for the different pipes and how the value for the same pipe varies across the cases. It can be observed from the plot that there isn’t much difference between case 1 and case 2 for timestep 2, a phenomenon that was also observed in timestep 7, as only case 2 and 3 are visible. As the DWEBC was normalised to obtain values between 0.1 and 1.0, case 3 resulted in solutions that are distinct and could be read easily, compared cases 1 and 2 as the values have a smaller range for meaningful conclusions to be made. This is also observed on graph plots presented in **Figs. 4, 5 and 6**, which illustrate the centrality difference on edges through colouring. The edges with higher DWEBC values are more prominent in case 3 for all the times as compared to case 1 and 2.

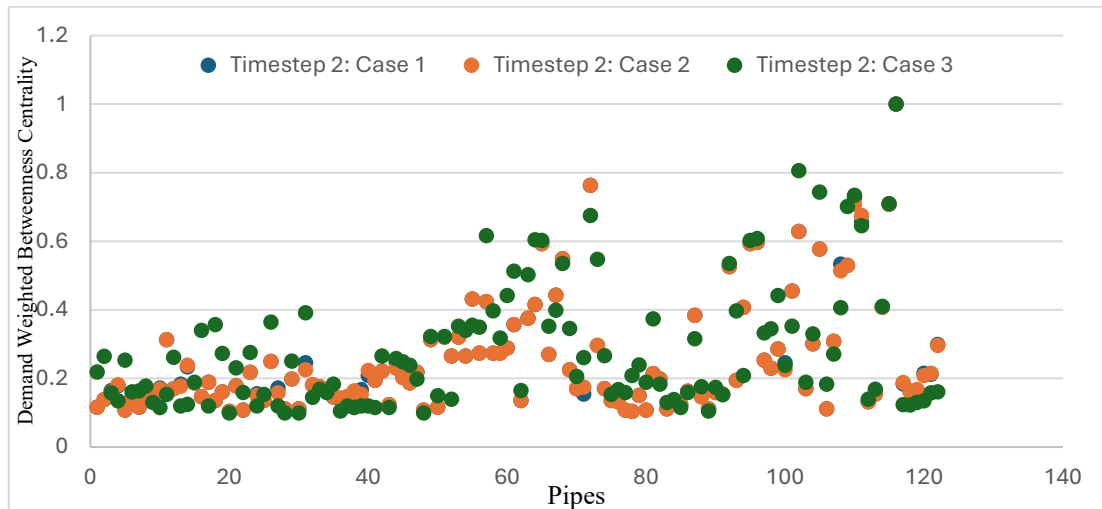


Fig 3. Kaina Laboratory Network DWEBC plots for Timestep 2.

From Fig. 4, there is insignificant differences between case 1 and case 2 plots in terms of DWEBC, and, with DWEBC values having a small range in either case, the difference in centrality difficult to ascertain in the plot, as compared to case 3 plot that has distinct values for the edges.

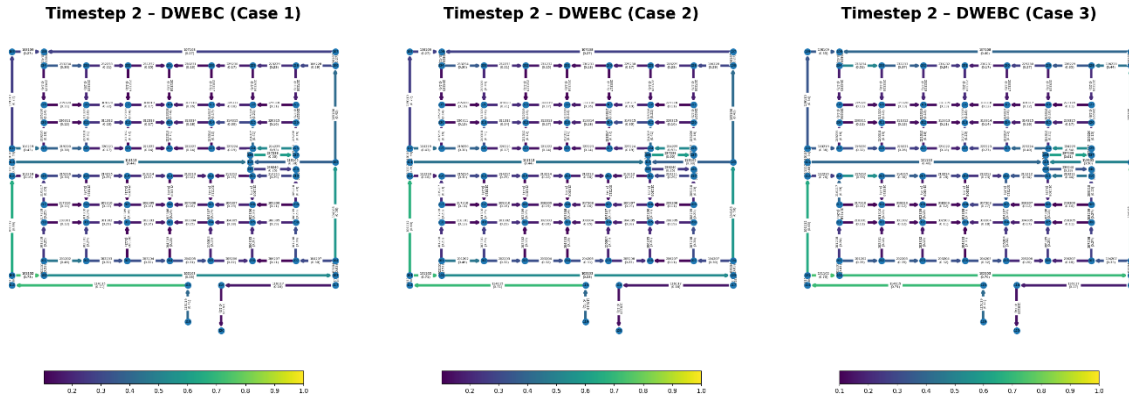


Fig. 4. Timestep 2 Cases 1, 2, and 3.

A similar result is observed for timestep 7 and 15 for the same network, in which edges central to the network are more pronounced in the graph plot, in addition to the differences between the edges DWEBC being noticeable for all pipes.

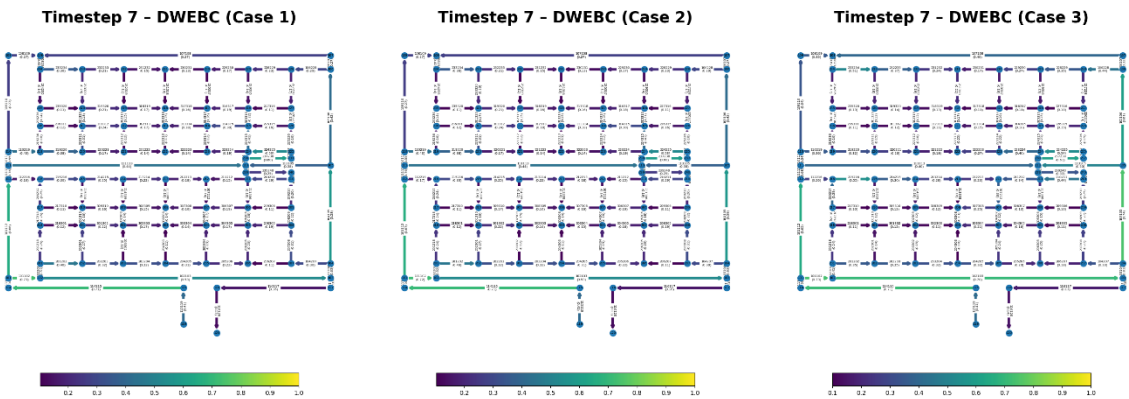


Fig. 5. Timestep 7 cases 1, 2, and 3.

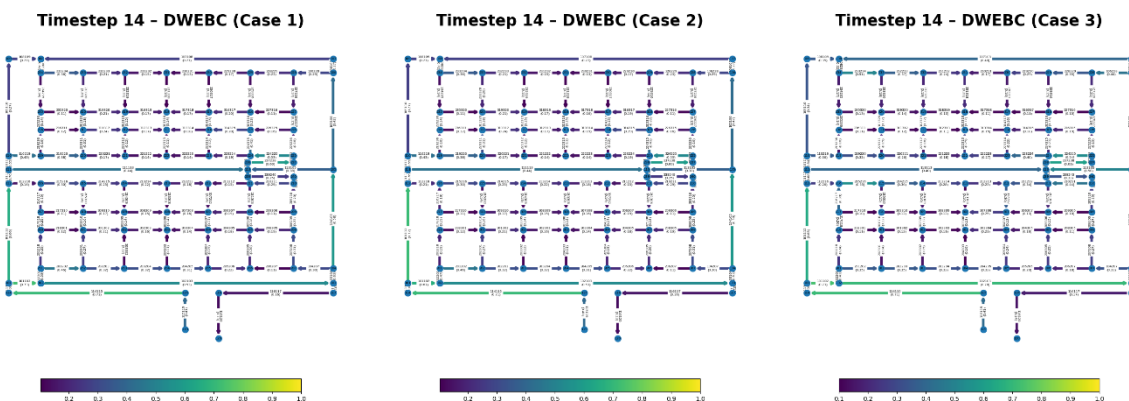


Fig. 6. Timestep 14 cases 1, 2, and 3.

Exnet Network

The algorithm was also evaluated on the Exnet benchmark network, presented in **Fig. 7**. This is a real, medium-size network consisting of 1,894 nodes, 2,471 pipes, 2 reservoirs, and 5 valves, however for this algorithm implementation adjustments were made as proposed by (LauCELLI et al., 2017) to ensure hydraulic consistent simulation of inlet flow using pressure driven analysis. In this case, the valves were replaced by pipes, five reservoirs were added as inflow nodes at: 3,003, 3,004, 3,005, 3,006, and 3,007, and the heads of all the new reservoirs and the existing reservoir 3,001 were set to 70 m. The elevations of junctions 1,107, 1,084, 726, 55, 41, 186, 1,092, 120, and 5,555 were fixed at 10 m. Such that we had 1884 nodes, 2465 pipes, and 7 reservoirs. The demand pattern given in **Fig. 2** is applied to the demand nodes.

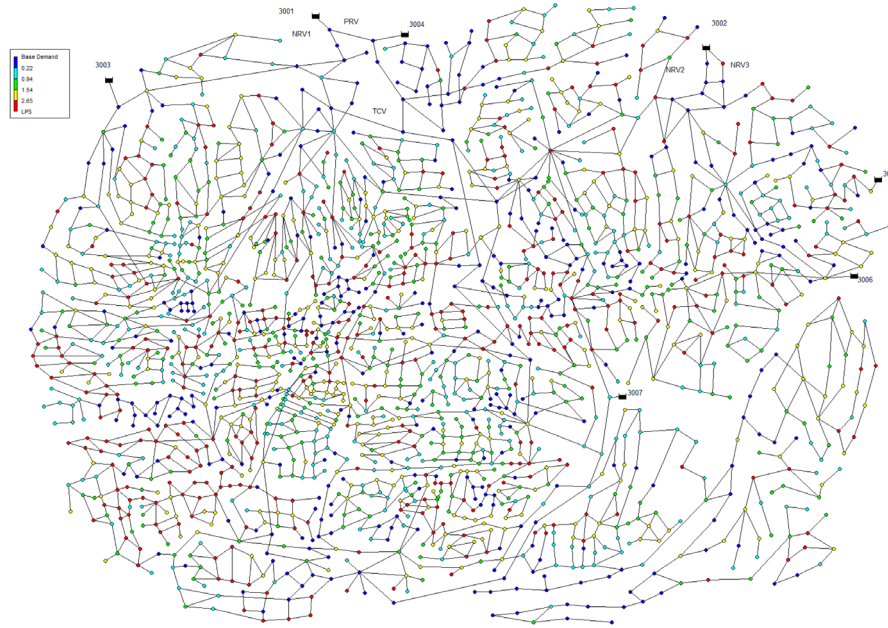


Fig. 7. Exnet raw water network layout.

The algorithm produced varying values for the DWEBC for the different timesteps for the different scaling factor and weight values. Selected timesteps have been provided in **Figs. 8, 9, and 10** to illustrate the centrality difference. The solutions presented, as for the Kaina network, have been selected to demonstrate performance for lowest demand (timestep 2), highest demand (timestep 7) and average demand (timestep 14). It was observed and can be demonstrated, as given in **Figs. 8, 9 and 10** that for the same edge/pipe the DWEBC value increases from case 1 to case 3 for timestep 2. When plotted, the most central pipes are visible on the network, with case 3 resulting in a more distinct alienation of the critical edges.

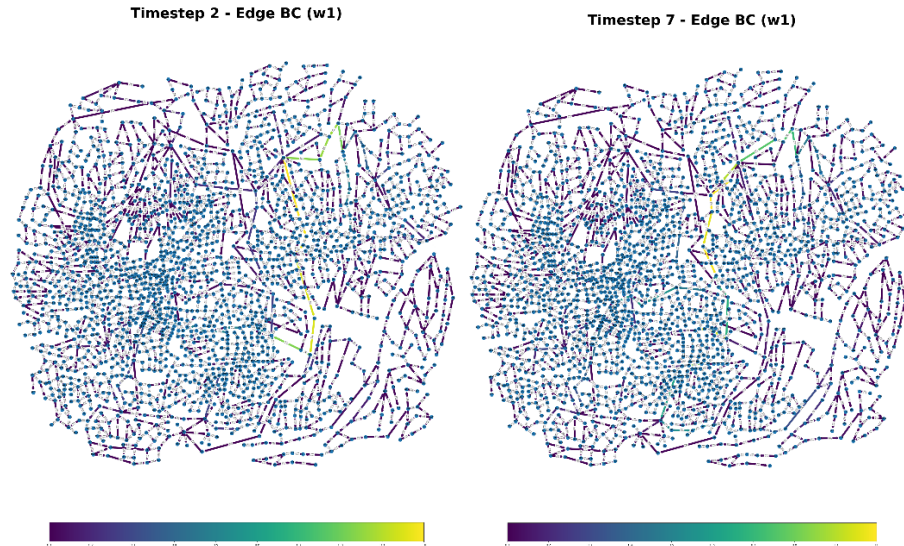


Fig. 8. Results for Case 1: (a) Timestep 2; (b) Timestep 7.

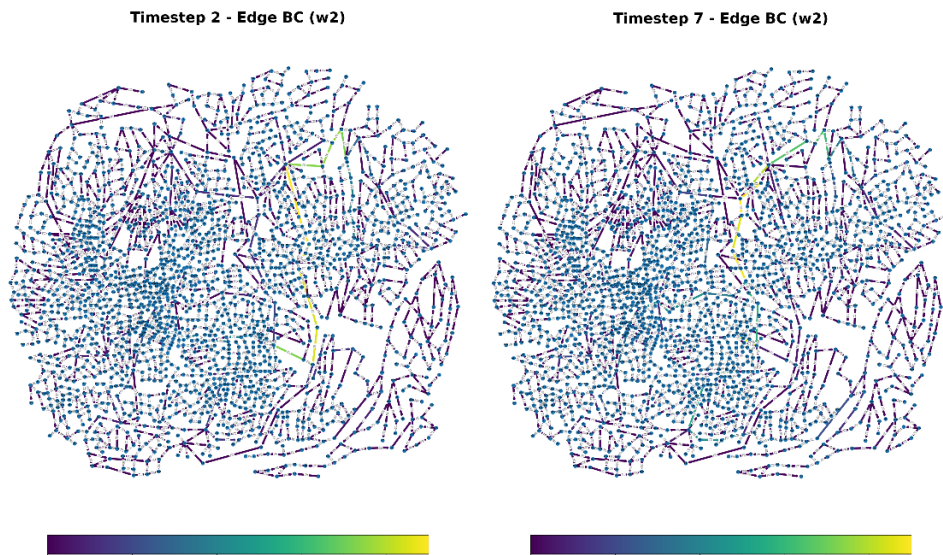


Fig. 9. Results for Case 2: (a) Timestep 2; (b) Timestep 7.

It can also be observed that case 3 can result in a completely different pipe being identified as having the highest DWEBC. Moving from **Figs. 8** to **Fig. 11**, different edges are marked with a higher centrality value. Interestingly so is the clarity in the network plot for case 3, suggesting a clear identification of edges that are critical for the network compared to case 1 and 2 tried, a scenario that was also observed in Kaina network.

Timestep 2 - Edge BC (w3)

Timestep 7 - Edge BC (w3)

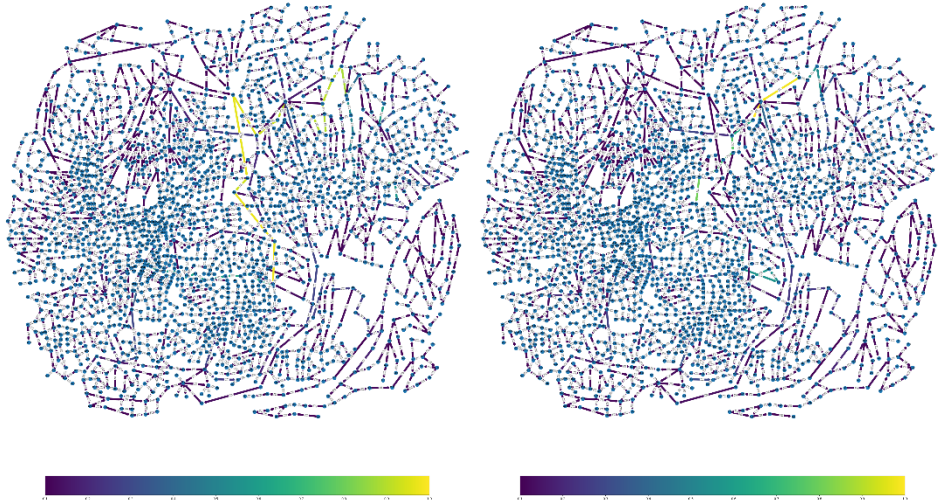
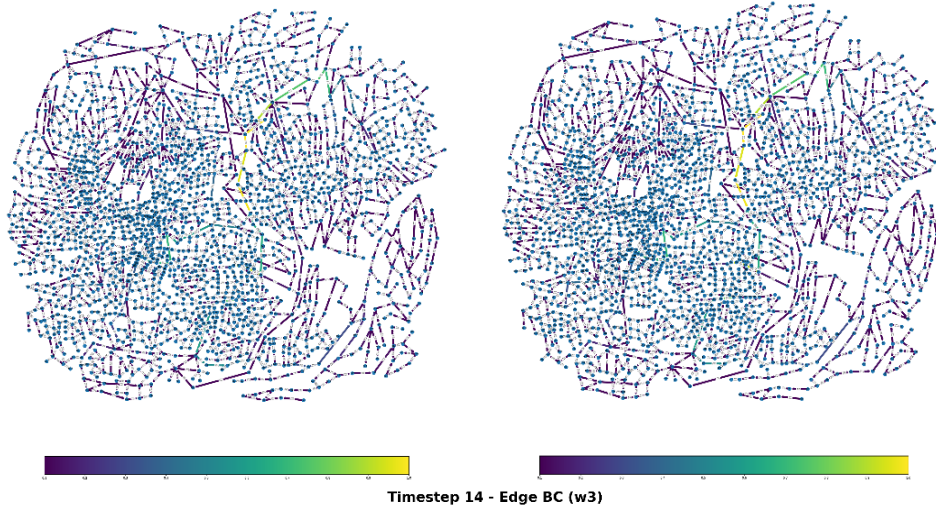


Fig. 10. DWEBC results for Case 3: (a) Timestep 2; (b) Timestep 7.

These results also suggest the procedure can be used to not only identify critical pipes but also identify timesteps that have the highest impact on determination of critical parts of the network.

Timestep 14 - Edge BC (w1)

Timestep 14 - Edge BC (w2)



Timestep 14 - Edge BC (w3)

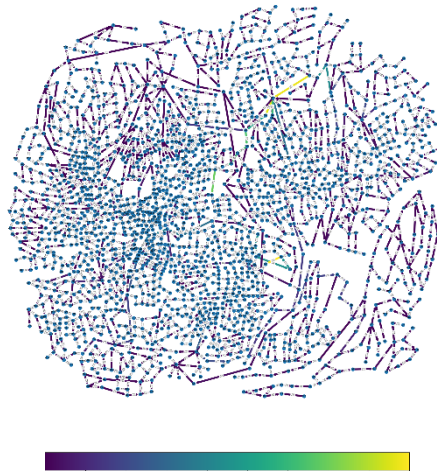


Fig. 11. DWEBC results for Timestep 14; (a) case 1 and 2; (b) case 3.

As observed in Fig 11. timestep 14 for all the three cases, the number of pipes with higher DWEBC is less compared to timestep 2 and 7. This is consisted with design principle as the network was designed to handle average demand, not the worst-case scenario.

Qualitative evaluation of DWEBC variants

When the three DWEBC variants are compared (**Fig. 12**), it is observed there is a strong agreement between case 1 and case 2. Case 1 uses a single global α across all pipes and all times, and Case 2 simply makes α time-varying (but still averaged over all pipes). As a result, their centrality scores end up being very tightly coupled. In practice this results scatter plots of weight1 vs weight2 forming a very narrow cloud, almost a straight line.

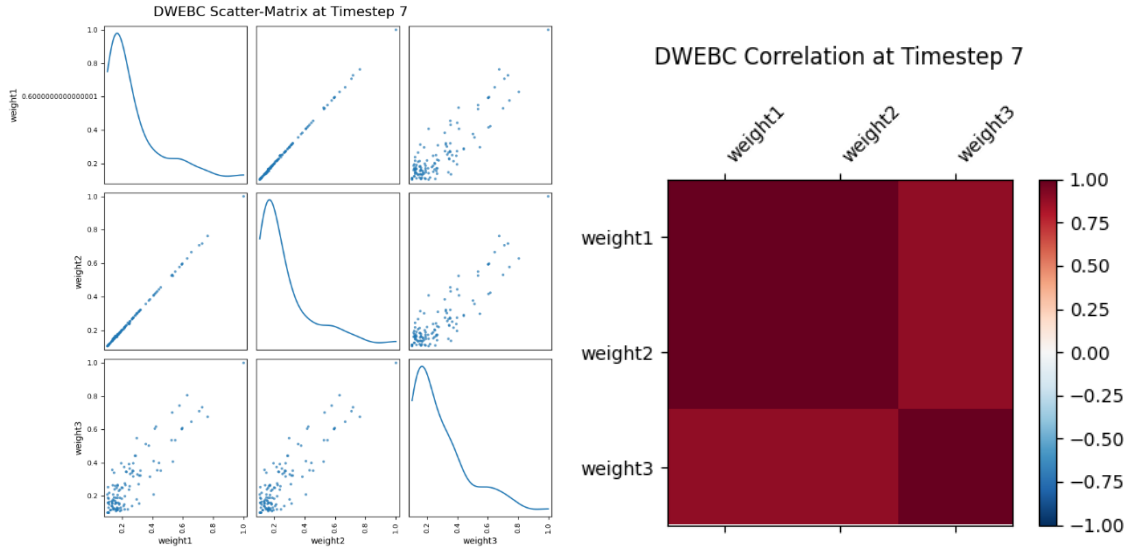


Fig. 12. DWEBC Scatter-Matrix (Left) and correlation (weight refers to case)(Right)

Correlation coefficients in heatmaps of these two columns are typically from 0.9–0.99 at all three timesteps considered because they are effectively the same ranking, just one is stretched or compressed a bit over time.

In contrast, case 3 diverges because it's fully local as it computes α per-pipe, per-timestep which injects spatial heterogeneity leading to pipes serving high-demand junctions having their centrality spike far more under case 3 than under case 1 and 2. It is noticed in the scatter plots that the cloud for case 1 against case 3 is much wider, with some edges falling well off the 45° line. Correlation between case 1 and case 3, and case 2 and case 3 often drops into the 0.6–0.8 range. This is the case because case 3 picks out those pipes that happen to sit “downstream” of peaks in local demand at each hour, whereas case 1 and case 2 smear every pipe with the same global average demand.

Considering temporal variation, across the three chosen timesteps, the low overall flow for timestep 2 means head loss and demand are small everywhere, so all three measures cluster more tightly with correlations ≥ 0.8 . The peak demand (timestep 7) means head loss and demand heterogeneity are largest, such that case 3 really pulls away from the global measures. This results in a scatter cloud that balloons out, and its correlation dip to the lowest point of the

day. The situation normalised a bit in the mid-afternoon (timestep 14, as the correlations bounce back towards timestep 2 values.

Spatial patterns can also be observed on the network maps when compared. Case 1 and Case 2 highlight essentially the same “backbone” of critical pipes, which are those long trunk pipes with consistently high flow and head loss. However, Case 3 captures the backbone of critical pipes considering the variability of properties across the network, hence even smaller lateral pipes feeding localized clusters of demand that get under-represented by the global averages, yet they serve high demand areas, are highlighted as well.

In conclusion, Case 1 and Case 2 are almost interchangeable, since one is just a time-stretched version of the other. While case 3 adds real edge-level distinction. It's less correlated with the global, especially at peak hours, because it zeros in on pipes serving high-demand nodes. This divergence is precisely why the full per-edge DWEBC is recommended as it exposes vulnerabilities that would be missed if only a single “average” network state is considered.

Practical Considerations of the Algorithm

The DWEBC algorithm that has been introduced in this study offers an option that is computationally powerful and efficient in identifying critical edges in WDNs. By combining local hydraulic stress indicators of head loss, demand, and pipe length, into a composite weight (W_1 , W_2 or W_3) and then computing edge-betweenness centrality over the resulting weighted graph, our method bridges the gap between purely topological metrics and full hydraulic simulations.

Balance of fidelity and efficiency

Traditional methods that apply the “remove-and-simulate” approach provide the most faithful picture of hydraulic vulnerability as they quantify exactly how delivered demand or nodal pressures change when a pipe breaks, however they require a separate simulation for each candidate edge which renders it computationally exhausting for very large or moderately sized networks. For a network of E pipes and T demand scenarios, with removal scales on the order of $O(E^2 \cdot T \cdot \text{sim})$, quickly becoming intractable even for moderately sized systems. In contrast, DWEBC runs in $O(T \cdot E + V \cdot E + V^2 \log V)$ time: the $O(T \cdot E)$ term comes from computing W for each timestep, and the graph-theoretic centrality cost, $O(VE + V^2 \log V)$, is handled by highly optimized C routines within NetworkX. For the Kaina case study of 122 pipes over 24 hours, the analysis completed in less than five seconds on a laptop equipped with an Intel 12th Gen Intel® Core™ i7-12700H 2.60 GHz processor, 32 GB RAM, and NVIDIA GeForce RTX 3060 6GB, in orders of magnitude faster than a corresponding suite of removal simulations.

Interpretability and practical relevance

EBC is a familiar concept to WDN analysts as it counts the fraction of shortest paths traversing each edge, emphasizing topological bottlenecks. By weighting paths according to W , we effectively tune those shortest paths to prefer hydraulically critical routes, resulting in edges that both carry substantial flow demand and lie on many such weighted paths naturally

surfacing at the top of the ranking. The color-coded network can be overlaid on GIS maps or EPANET schematics, which makes this blend of topology and hydraulics output immediately understandable to engineers and network operators hence facilitating rapid situational awareness and control.

Comparative strengths

When compared to alternative metrics, our method occupies a compelling middle ground. While the Max-flow/Min-cut sensitivity offers physically rigorous capacity analysis, it still demands solving a full network flow for each edge, and it has a complexity of $O(E^2\sqrt{V})$. The Spectral-gap, algebraic connectivity, on the other hand excels at capturing global connectivity loss via Laplacian eigenvalues in $O(V^2)$ time up front, then $O(1)$ per edge. However, it ignores hydraulic loads and cannot distinguish high-flow pipes from lightly utilized ones, a phenomenon the DWEBC incorporates through the target demand. The WNTR resilience metrics provide time-series demand-deficit indices under break scenarios but require multiple EPANET runs and substantial memory to store simulation states, hence computationally exhaustive as well. The Flow-based centrality is Linear in $T \cdot E$ and trivial to compute. Aside this, it lacks any global perspective on the entire network. We also compared the DWEBC against WFEBC by (Ulusoy et al., 2018) on the Net3 benchmark network. For our case, the obvious critical pipes are removed from the analysis (those connected to tanks and reservoirs). It can be observed that DWEBC using case 3 is better at delineating the criticality of the pipes in the critical corridor.

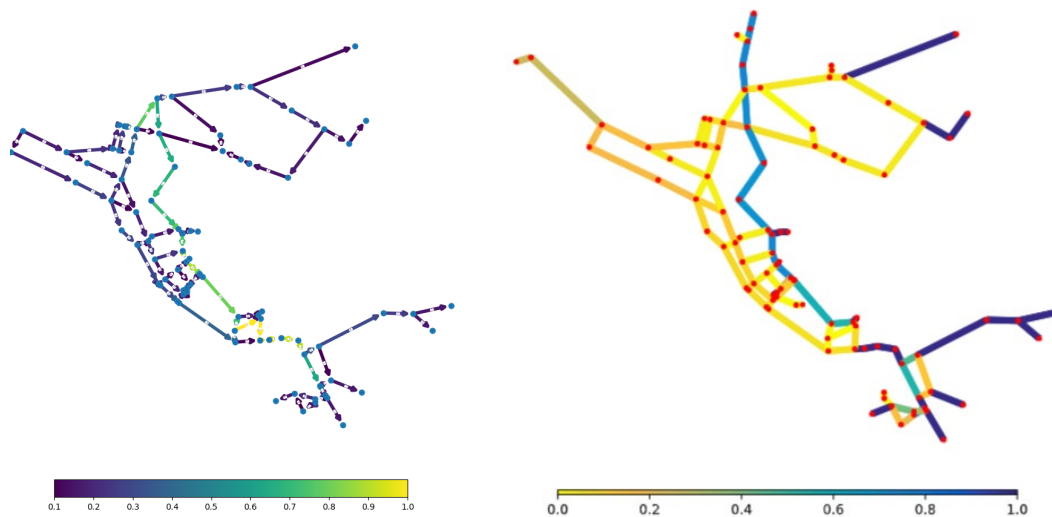


Fig 13. (a) DWEBC versus (b) WFEBC of Net3. ‘The betweenness of each link is weighted by customer demand and normalised by the maximum total flow it would have to support if there were no alternative paths to yield a measure of WFEBC. The darker the link, the more critical’(Ulusoy et al., 2018).

The DWEBC approach combines the global path-based insights of centrality measures with domain-specific hydraulic weighting. This yields high relevance in the output without excessive computational or memory burdens.

Deployment and future directions

The DWEBC runs in seconds for networks of several thousand pipes and dozens of timesteps, making it convenient for water utilities to embed in daily or weekly resilience assessments of WDNs. And as demand patterns shift, due to seasonal variation, new developments, or emergency events, the analysis can quickly re-rank critical links, guiding sensor placement or preventive maintenance.

The composite weight used in DWEBC can be adapted and calibrated. Future work might incorporate real-time flow telemetry, such metrics as nodal demand and water age, into the local stress calculation, further aligning the ranking with observed network performance. Also considering a two-stage resilience framework, the DWEBC can also be used to generate a short-list of high-risk pipes (e.g., top 5–10%) on which detailed hydraulic simulations can be performed by either removal or pressure-deficit scenarios to quantify resilience impacts. This hybrid approach would retain computational tractability while preserving the rigor of full hydraulic modelling where it matters most. These suggestions mean the algorithm can be employed within a Digital Twin (DT) framework that would support real-time and realistic network analysis and control.

Conclusion

This study has developed and demonstrated Demand Weighted Edge Betweenness Centrality (DWEBC) as a practical metric for identifying hydraulically critical pipes in water distribution networks for vulnerability assessment. In embedding nodal demand and head loss into the traditional betweenness centrality, DWEBC identifies pipes that, if disrupted, would separate high demand corridors, an outcome not captured by topology-only measures. Application to both the Kaina laboratory network and the Exnet benchmark network confirmed that the pipe-specific scaling variant provides the most distinct identification of critical edges, while retaining execution times that are suitable for routine utility operations.

Compared to existing approaches, ranging from exhaustive “remove-and-simulate” analyses to spectral-gap and flow-based centralities, DWEBC strikes a balance between computational efficiency and hydraulic fidelity. As observed, its outputs can be seamlessly overlaid on GIS or EPANET schematics, supporting rapid situational awareness, preventive maintenance targeting, and adaptive resilience workflows.

However, the study is limited by DWEBC reliance on static demand pattern, it remains an approximation as it does not directly model pressure propagation or network feedback under pipe failure. To address this, future research should integrate real-time flow telemetry and pressure feedback into the weighting function, in which demand fluctuates over time, enabling truly dynamic DWEBC assessments. For the most critical few pipelines, to avoid targeted and many EPANET simulation or max-flow analysis, incorporation of the suggested real-time flow telemetry would guarantee the realistic representation as it combines the strengths of both surrogate and rigours methods. In doing so, DWEBC can serve as a cornerstone of next-generation WDN monitoring and planning strategies, ensuring that high-demand areas receive prioritized attention in the face of aging infrastructure and evolving urban stresses.

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